Mixed Tukey Exponentially Weighted Moving Average-Modified Exponentially Weighted Moving Average Control Chart for Process Monitoring

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Abstract

The goal of this study is to present the mixed Tukey exponentially weighted moving average-modified exponentially weighted moving average control chart (MEME-TCC) for monitoring process location with symmetric and skewed distributions in an attempt to significantly improve detection ability. With the benefits of nonparametric assumption robustness. The average and median run lengths are supporting measurements for assessing the performance of a monitoring scheme using Monte Carlo simulation. Furthermore, the average extra quadratic loss (AEQL), relative mean index (RMI), and performance comparison index (PCI) can all be used to evaluate overall performance criteria. The proposed chart is compared with existing charts such as; EWMA, MEWMA, TCC, MEME, MMEE, and MMEE-TCC. The comparison result shows that the proposed chart is the best control chart for detecting small to moderate shifts among all distributional settings. Nevertheless, the EWMA chart detects large shifts more effectively than other charts, except in the case of the gamma distribution, where MEWMA performs best. The results of adapting the proposed control chart to two sets of real data corresponded to the research findings.

1- Introduction

Statistical methods are employed to identify and comprehend variability, such that resultant observations of a process or occurrence do not create the same outcome. We all experience variability in our ordinary routines, and statistical thinking can help us incorporate that variability into our judgment processes. The control chart is an effective Statistical Process Control (SPC) tool for overseeing workflow by recognizing and resolving issues as they happen. Generally, the Shewhart, exponentially weighted moving average (EWMA), moving average (MA), and cumulative sum (CUSUM) control charts are indeed the most prevalent. The well-known Shewhart control chart [1] can be used to detect large changes in operating parameters, while the EWMA [2], MA [3], and CUSUM control charts [4] can detect minor to moderate changes in the parameters of interest. A modified exponentially weighted moving average (MEWMA) control chart [5] was recently developed to detect tiny changes more quickly. However, many researchers believe that a mixed control chart can increase the effectiveness of a control chart. These would be parametric control charts that assume normality. In this regard, Abbas et al. [6] presented a mixed EWMA-CUSUM (MEC) control chart for the monitoring procedure; they compared the proposed chart to other charts (EWMA, CUSUM, FIR CUSUM, and FIR EWMA) and discovered that the proposed chart was more sensitive to detect small shifts, whereas Zaman et al. [7] developed the mixed CUSUM-EWMA control chart (MCE) and observed that, when compared with existing charts, it is very sensitive for detecting small to moderate shifts.

An EWMA control chart based on moving average statistics for exponentially distributed quality was created by Khan et al. [8] and compared their average run length with other charts. The MA-EWMA control chart was proposed by Taboran et al. [9], while Sukparungsee et al. [10] envisioned a mixed EWMA-MA control chart and evaluated its efficiency by using the average run length, median run length, and standard deviation of run length calculated via other charts. A new mixed EWMA–progressive mean (MEP) chart was created by Abbas et al. [11] and used average run length to compare performance. Both mixed homogeneously weighted moving average and cumulative sum (HWMA-CUSUM) and CUSUM-HWMA control charts were presented by Abid et al. [12, 13], and the results show that the designed chart performs better than the existing charts. Similarly, Saengsura et al. [14] and Talordphop et al. [15] proposed mixed MA-CUSUM and MEWMA-MA (MMEM) control charts for monitoring process changes, respectively. Their results show that the proposed chart was more efficient than the existing control chart.

Data from authentic processes, such as economics, healthcare, industry, and the environment, are no longer premised on the assumption of normalcy. Nonparametric control charts are an appropriate option for using control charts that do not make normal assumptions. Tukey's control chart (TCC) was developed by Alemi [16] for ease of use in non-normal occurrences and when the workflow distribution is uncertain, and it is effective in monitoring the mean process. However, Sukparungsee [17] also discussed the robustness of Tukey's control charts, finding that the asymmetric Tukey's control chart outperforms the symmetric Tukey's control chart in both cases of skew and non-skew procedures. Furthermore, several authors combine the efficiency of TCC with other control charts, such as the EWMA-TCC [18], MEC-TCC [19], Tukey MA-EWMA and Tukey MA-DEWMA [20, 21], and Tukey MEWMA-MA [22], to establish control charts that can respond to changes quickly and apply to a variety of circumstances with the smallest constrictions.

Motivated by the impressive talents of EWMA, MEWMA, and TCC charts, we attempted to combine their features and propose a more efficient mixed Tukey EWMA-MEWMA (MEME-TCC) chart for process location under symmetric and asymmetric distributions. Average run length (ARL) and median run length (MRL) are commonly used as performance measures in Monte Carlo simulation. Furthermore, the average extra quadratic loss (AEQL), relative mean index (RMI), and performance comparison index (PCI) can all be used to evaluate overall performance criteria. The EWMA, MEWMA, TCC, Mixed Exponentially Weighted Moving Average - Modified Exponentially Weighted Moving Average (MEME), Mixed Modified Exponentially Weighted Moving Average - Exponentially Weighted Moving Average (MMEE), and MMEE-TCC charts are employed for the comparison. Furthermore, the stated MEME-TCC chart is accomplished in a realistic scenario to exemplify its practical significance.

2- Structure of Control Charts

Suppose X as the process variable following a normal distribution with mean μ and variance σ^2. Take a random sample $X_j, j = 1, 2, \ldots$ is j^{th} independent and identically distributed observations. Using this relevant information, EWMA, MEWMA, TCC, MEME, MMEE, MMEE-TCC, and proposed (MEME-TCC) charts are explained below.

2-1- Exponentially Weighted Moving Average (EWMA) Control Chart

Robert [2] pioneered the EWMA structure to monitor the process mean through plotting the below EWMA statistic:

$$Z_j = \lambda X_j + (1 - \lambda)Z_{j-1}; \quad j = 1, 2, \ldots \tag{1}$$

where λ is the weighing parameter, such that $0 < \lambda \leq 1$ and. The mean and variance of Z_j are given as follows:

$$E(Z_j) = \mu \tag{2}$$

$$V(Z_j) = \sigma^2 \left[\frac{\lambda}{2 - \lambda} \left(1 - (1 - \lambda)^{2t})\right) \right]. \tag{3}$$

When $j \to \infty$, the asymptotic variance is;

$$V(Z) = \sigma^2 \left(\frac{\lambda}{2 - \lambda} \right). \tag{4}$$

The upper and lower control limits of EWMA chart are:

$$UCL/LCL = \mu \pm V_1 \sigma \sqrt{\left(\frac{\lambda}{2 - \lambda} \right)} \tag{5}$$

where V_1 is the control limits coefficient selected to achieve ARL_0. μ is the mean of the process and σ^2 is variance of the process.

2-2- Modified Exponentially Weighted Moving Average (MEWMA) Control Chart

Khan et al. [5] presented the MEWMA control chart for detecting both small and large shifts in the process mean. The charting statistic of MEWMA control chart having a smoothing parameter $\lambda(0 < \lambda \leq 1)$ can be defined as:

$$M_j = \lambda X_j + (1 - \lambda)M_{j-1} + k(X_j - X_{j-1}) \tag{6}$$

However, the constant $k \neq 0$ can be chosen independently of λ and we used $k = -\lambda/2$ to minimize the asymptotic variance of the MEWMA chart. The mean and asymptotic variance of M_j are given as follows:

$$E(M_j) = \mu \tag{7}$$

$$V(M_j) = \sigma^2 \left[\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda} \right] \tag{8}$$

The upper and lower control limits of MEWMA chart are:

$$UCL/LCL = \mu \pm V_2 \sigma \sqrt{\frac{(\lambda + 2\lambda k + 2k^2)}{(2 - \lambda)}} \tag{9}$$

where V_2 the control limits coefficient selected to achieve ARL_0. μ is the mean of the process and σ^2 is variance of the process.

2-3- Tukey's Control Chart (TCC)

The TCC is the nonparametric control chart. The control limits are:

$$UCL = Q_3 + V_3(IQR)$$
$$LCL = Q_1 - V_3(IQR) \tag{10}$$

where IQR is the interquartile range $(Q_3 - Q_1)$, Q_1 and Q_3 are the first and the third quartiles and V_3 is the control limits coefficient for the TCC.

2-4- Mixed Exponentially Weighted Moving Average - Modified Exponentially Weighted Moving Average (MEME) Control Chart

The MEME chart is a combination of EWMA and MEWMA control chart. The statistic of MEME control chart is defined as:

$$Z_{MEME_j} = \lambda M_j + (1 - \lambda)Z_{MEME_{j-1}}; \quad j = 1, 2, \ldots \tag{11}$$

where $M_j = \lambda X_j + (1 - \lambda)M_{j-1} + k(X_j - X_{j-1})$, such that $0 < \lambda \leq 1$.

The upper and lower control limits of the MEME chart are given as follow:

$$UCL/LCL = \mu_M \pm V_4 \sigma_M \sqrt{\left(\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda} \right)} \tag{12}$$

where V_4 the control limits coefficient selected to achieve ARL_0. μ_M is the mean of MEWMA and σ_M is the standard deviation of MEWMA.

2-5- Mixed Modified Exponentially Weighted Moving Average - Exponentially Weighted Moving Average (MMEE) Control Chart

The MMEE chart is a combination of MEWMA and EWMA control chart. The statistic of MMEE control chart is defined as:

$$M_{MMEE_j} = \lambda Z_j + (1 - \lambda)M_{MMEE_{j-1}} + k(Z_j - Z_{j-1}); \quad j = 1, 2, \ldots \tag{13}$$

where Z_j is the statistic of EWMA: $Z_j = \lambda X_j + (1 - \lambda)Z_{j-1}$.

The upper and lower control limits of the MMEE chart are given as follow:

$$UCL/LCL = \mu_Z \pm V_5 \sigma_z \sqrt{\left(\frac{\lambda}{2 - \lambda} \right)} \tag{14}$$

where V_5 the control limits coefficient selected to achieve ARL_0. μ_Z is the mean of EWMA and σ_Z is the standard deviation of EWMA.

2-6- The proposed Mixed Tukey Exponentially Weighted Moving Average - Modified Exponentially Weighted Moving Average (MEME-TCC) Control Chart

The MEME-TCC control chart combines the MEME and TCC control charts and employs the MEME statistic. The upper and lower control limits of the MEME-TCC chart are given as follow:

$$
\begin{aligned}
UCL &= Q_3 + V_6(IQR)\sqrt{\left(\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda}\right)} \\
LCL &= Q_1 - V_6(IQR)\sqrt{\left(\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda}\right)}
\end{aligned}
\tag{15}
$$

where V_6 is a coefficient of the control limits of the MEME-TCC control chart. λ is the weighing parameter, such that $0 < \lambda \leq 1$. IQR is the inter quartile range, Q_1 and Q_3 are the first and third quartiles.

2-7- Mixed Tukey Modified Exponentially Weighted Moving Average - Exponentially Weighted Moving Average (MMEE-TCC) Control Chart

Likewise, MMEE-TCC is the combination of MMEE and TCC, which uses the statistic of MMEE. The upper and lower control limits of the MMEE-TCC chart are given as follow:

$$
\begin{aligned}
UCL &= Q_3 + V_7(IQR)\sqrt{\left(\frac{\lambda}{2 - \lambda}\right)} \\
LCL &= Q_1 - V_7(IQR)\sqrt{\left(\frac{\lambda}{2 - \lambda}\right)}
\end{aligned}
\tag{16}
$$

where V_7 is a coefficient of the control limits of the MMEE-TCC control chart. λ is the weighing parameter, such that $0 < \lambda \leq 1$. IQR is the inter quartile range, Q_1 and Q_3 are the first and third quartiles.

3- Performance Measures and Optimization Criteria

A run is a collection of data points mapped on a control chart until one of them shows signs of an out-of-control signal, the number of points in a run referring to the run length (RL). The most ubiquitously used criterion for assessing a control chart's ability to detect specific shifts is average run length (ARL). This is described as the expected number of signals that should be mapped prior to the appearance of an out-of-control sensor. When the procedure is still under control, ARL_0 emerges, while ARL_1 emerges when the procedure is out-of-control. It's really desirable to detect a change in the process as soon as possible, which means that ARL_1 should be small in order to ensure that the control chart is effective. Median run length (MRL) is an additional supporting measure for evaluating the performance of a monitoring scheme. The ARL and MRL formulas are defined as:

$$
ARL = \frac{\sum_{j=1}^{N} RL_j}{N}
\tag{17}
$$

$$
MRL = Median(RL)
\tag{18}
$$

Therefore, the average extra quadratic loss (AEQL) can assess overall performance all over the process shift range $(\delta_{max} - \delta_{min})$ and is widespread seen as a potential determinant by many researchers. As a result, the chart with the lowest AEQL value is deemed the most efficient. Likewise, a tiny relative mean index (RMI), as shown in the chart, has a quick detection capability overall [23]. The AEQL formula is defined as

$$
AEQL = \frac{1}{\delta_{max} - \delta_{min}} \sum_{\delta_{min}}^{\delta_{max}} \delta^2 \times ARL(\delta)
\tag{19}
$$

where δ_{max} and δ_{min} represent the lower and upper bounds of the shift, respectively. δ denotes the amount of change in the process mean through standard deviations, $ARL(\delta)$ represents the ARL value of a chart for the given shift.

The RMI attribute is derived as follows:

$$
RMI = \frac{1}{n} \sum_{i=1}^{n} \frac{ARL(\delta_i) - ARL*(\delta_i)}{ARL*(\delta_i)}
\tag{20}
$$

where n refers to the number of shifts that are considered, $ARL(\delta_i)$ represents the ARL value of a chart for the given shift and $ARL*(\delta_i)$ is the lowest ARL value across all competing charts for the given shift.

In addition, the performance comparison index (PCI) is the fraction between the AEQL of a chart and the AEQL of the best chart under similar condition [24], where the PCI value of the most efficient chart is one and the PCI value of the other competitive charts is greater than one. The PCI value is provided by

$$PCI = \frac{AEQL}{AEQL_{lowest}}$$
(21)

To find solutions, use the procedure depicted in Figure 1.

```
                        ┌──────────┐
                        │  Start   │
                        └──────────┘
                             │
    ┌────────────────────────────────────────────────────────┐
    │  Generate a random sample of size n from any certain distribution.  │
    └────────────────────────────────────────────────────────┘
                             │
         ┌──────────────────────────────────────────┐
         │  Consider V at a specified value of ARL₀ = 370  │
         └──────────────────────────────────────────┘
                             │
         ┌──────────────────────────────────────────┐
         │  Compute the control limit (UCL/LCL) and the statistic of the
            control charts.  │
         └──────────────────────────────────────────┘
                             │
              ◇ Iterate 200,000 repetitions (N) ◇  —— No
                             │ Yes
         ┌──────────────────────────────────────────┐
         │  Use this run length values to calculate the performance
            measure of the chart (ARL, MRL)  │
         └──────────────────────────────────────────┘
                             │
              ◇ Complete Shift ◇ —— No
                             │ Yes
                        ┌──────────┐
                        │   End    │
                        └──────────┘
```
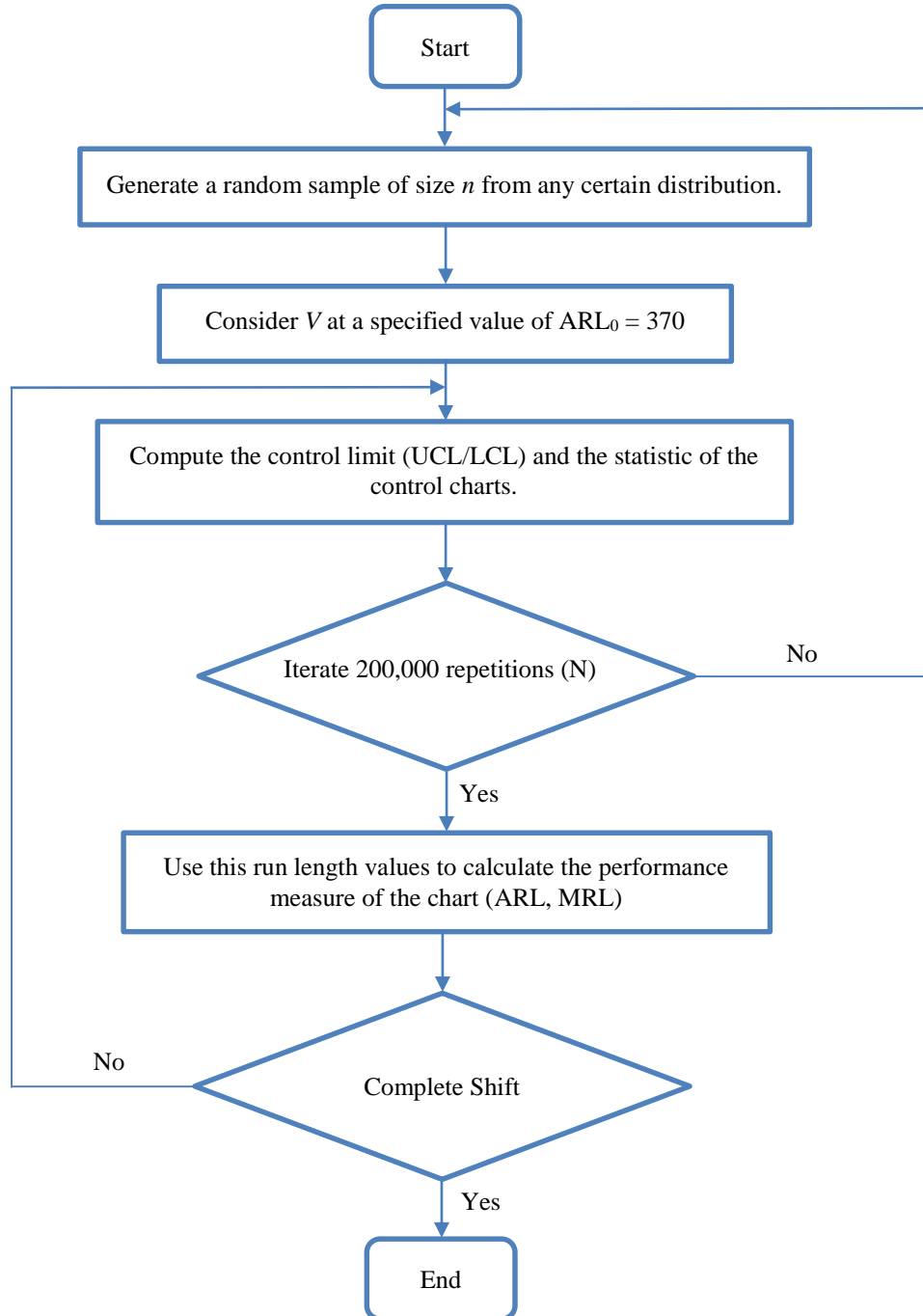
Figure 1. The flowchart of the process for determining performance measures

4- Performance Comparisons

This section provides the proposed chart evaluation based on the previously described performance indicators from 10,000 sample sizes (n) and 200,000 repetitions (N) in Monte Carlo simulations under ARL$_0$ = 370. The run length features of all the charts are acquired through simulations with $\lambda = 0.25$, shifts (0, $\pm0.05, \pm0.10, \pm0.25, \pm0.50, \pm0.75, \pm1.00, \pm1.50, \pm2.00, \pm3.00, \pm4.00$) and $k = -0.125$, for the normal(0,1), Laplace(0,1), Exponential(1), and

gamma(4,1) distributions. Besides that, we compare the performance of the proposed chart (MEME-TCC) with that of existing charts, and the chart with the lowest ARL_1 is declared to be the best.

From the simulation, the control limit constants of mixed nonparametric control charts (MEME-TCC and MMEE-TCC) are quite high when compared to their mixed parametric control charts (MEME and MMEE) in all distributions. The findings overall under normal distribution presented in Table 1 actually indicate from the ARL profile that the proposed chart (MEME-TCC) has slightly stronger detectability in small to moderate shifts (0.05 to 1.00), although the EWMA control chart outperforms in large shifts (1.50 to 4.00). The proposed charting performs far better than MEWMA at shifts 0.05 to 1.50 and far better than TCC at shifts 3.00 to 4.00.

Table 1. The ARL and MRL evaluations of the proposed chart and existing control charts under normal distribution

Shift	EWMA $V_1 = 2.899$		MEWMA $V_2 = 2.199$		TCC $V_3 = 1.723$		MEME $V_4 = 1.953$		MMEE $V_5 = 1.827$		MEME-TCC $V_6 = 2.682$		MMEE-TCC $V_7 = 2.264$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
-4.00	**0.66**	1	1.07	1	1.17	1	2.13	2	2.13	2	1.48	1	2.11	2
-3.00	**1.19**	1	1.49	1	1.93	1	2.76	3	2.75	3	2.07	2	2.72	3
-2.00	**2.46**	2	2.69	2	4.87	3	3.95	4	3.94	4	3.06	3	3.89	4
-1.50	**4.17**	4	4.28	4	12.69	9	5.29	5	5.30	5	4.25	4	5.22	5
-1.00	11.84	7	9.10	7	38.39	27	8.99	8	9.00	8	**6.18**	**6**	8.86	8
-0.75	21.64	13	16.41	13	71.18	49	14.37	12	14.39	12	**9.94**	**10**	14.11	12
-0.50	43.25	29	38.44	28	135.37	94	30.38	23	30.47	23	**20.14**	**21**	29.52	22
-0.25	137.69	94	129.24	90	246.39	171	103.29	73	103.51	74	**61.58**	**60**	99.02	71
-0.1	302.93	205	290.96	203	314.02	217	265.66	186	266.85	187	**202.73**	**169**	249.09	174
-0.05	348.72	242	347.39	242	327.34	226	337.31	236	338.42	238	**255.86**	**209**	320.33	224
0	370.52	257	370.67	258	370.22	255	370.42	258	370.43	258	**370.21**	258	370.25	258
0.05	347.79	242	346.66	240	332.16	230	336.26	234	338.07	236	**255.73**	**191**	321.4	238
0.1	302.6	204	289.90	201	321.87	223	265.99	186	266.69	186	**201.56**	**154**	250.81	193
0.25	138.14	95	129.30	91	261.19	181	103.69	74	103.79	74	**60.59**	**60**	100.29	77
0.50	43.2	29	38.37	28	146.79	102	30.37	23	30.52	23	**20.54**	**21**	30.24	24
0.75	21.99	13	16.38	13	76.62	53	14.37	12	14.39	12	**9.6**	**10**	14.65	12
1.00	11.85	7	9.05	7	41.40	28	9.01	8	9.02	8	**6.22**	**6**	9.13	8
1.50	**4.18**	4	4.29	4	13.51	9	5.30	5	5.30	5	4.21	4	5.36	5
2.00	**2.47**	2	2.69	2	5.14	3	3.94	4	3.94	4	3.07	3	3.97	4
3.00	**1.19**	1	1.49	1	1.98	1	2.76	3	2.75	3	2.08	2	2.72	3
4.00	**0.67**	1	1.07	1	1.19	1	2.13	2	2.13	2	1.49	1	2.14	2

Note: The fewest numbers of and MRL appear in bold.

The overall results of the Laplace distribution shown in Table 2 demonstrate that the proposed chart performs better than its competitors, namely EWMA, MEWMA, TCC, MEME, MMEE, and MMEE-TCC, for shifts ranging from 0.05 to 2.00, whereas the EWMA chart is the best for shifts 3.00 and 4.00. Similarly, when compared to MEWMA, the proposed chart outperforms in shifts ranging from 0.05 to 2.00, and when compared to TCC, the proposed chart performs better in all shifts. However, positive shifts have similar outcomes as negative shifts.

In addition, we reveal the outcomes of the control chart with skew distributions. Table 3 demonstrates the exponential distribution. The proposed chart has marginally stronger detection performance in shifts 0.05 to 2.00, but the EWMA chart outshines in shifts 3.00 and 4.00. When compared to MEWMA and TCC charts, the proposed chart detects shifts from 0.05 to 2.00 more quickly. The Gamma distribution results in Table 4 indicate that the proposed chart's ARL_1 is fewer than the other charts in shifts 0.05 to 0.75, while the MEWMA chart is the greatest in shifts 1.00 to 4.00. At shifts 0.05 to 2.00, the proposed charting outperforms EWMA, and it outperforms TCC at all shifts.

Table 2. The ARL and MRL evaluations of the proposed chart and existing control charts under Laplace distribution

Shift	EWMA $V_1 = 2.357$		MEWMA $V_2 = 1.749$		TCC $V_3 = 3.767$		MEME $V_4 = 1.432$		MMEE $V_5 = 1.340$		MEME-TCC $V_6 = 3.136$		MMEE-TCC $V_7 = 2.717$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
-4.00	**1.65**	2	1.91	2	12.88	8	3.00	2	3.00	3	2.99	3	3.01	3
-3.00	**2.81**	3	2.88	3	36.56	24	3.85	3	3.85	4	3.83	4	3.86	4
-2.00	6.60	6	5.93	5	99.23	68	5.87	4	5.88	5	**5.84**	5	5.88	5
-1.50	13.31	11	11.28	9	108.87	99	8.61	5	8.62	8	**8.55**	8	8.64	8
-1.00	36.73	27	30.39	22	145.52	132	17.54	8	17.55	14	**17.35**	14	17.59	14
-0.75	69.33	49	58.44	42	178.92	162	31.38	12	31.39	24	**31.09**	**23**	31.61	24
-0.50	139.43	98	122.72	87	263.34	179	70.02	23	70.20	51	**68.77**	**50**	70.68	51
-0.25	270.48	187	255.95	178	305.12	204	190.29	73	190.99	134	**185.81**	**130**	192.15	135
-0.1	349.86	243	346.17	240	350.78	217	323.47	187	324.67	227	**315.27**	**219**	320.93	224
-0.05	364.58	253	363.77	252	360.45	228	355.72	236	357.29	249	**339.67**	**243**	351.64	246
0	370.75	257	370.56	257	370.26	255	370.74	258	370.81	258	370.11	258	370.22	258
0.05	364.82	253	364.27	253	358.53	227	356.07	234	356.94	249	**328.48**	**242**	346.31	243
0.1	349.52	242	346.95	241	349.56	218	321.90	186	324.61	226	**299.02**	**216**	321.65	221
0.25	269.52	188	255.68	179	307.95	207	190.79	74	190.59	134	**187.94**	**129**	188.88	132
0.50	139.00	97	122.73	86	259.34	178	69.94	23	70.04	51	**67.59**	**49**	69.52	50
0.75	69.16	49	58.39	42	176.35	161	31.42	12	31.51	24	**30.46**	**23**	31.10	24
1.00	36.66	27	30.42	22	146.36	133	17.53	8	17.56	14	**17.16**	14	17.39	14
1.50	13.35	11	11.29	9	109.02	100	8.62	7	8.63	8	**8.51**	7	8.59	7
2.00	6.58	6	5.95	5	99.36	69	5.87	5	5.88	5	**5.83**	5	5.86	5
3.00	**2.82**	3	2.88	3	36.87	25	3.85	3	3.85	4	3.89	4	5.89	4
4.00	**1.64**	2	1.92	2	12.92	9	3.00	2	3.00	3	2.99	3	2.99	3

Note: The fewest numbers of and MRL appear in bold.

When compared to the mixed parametric chart, the proposed chart can detect all shifts quickly in all distributions. It is possible to conclude that the proposed chart detects small to moderate shifts in the process more quickly than the single control chart and mixed parametric control charts.

Table 3. The ARL and MRL evaluations of the proposed chart and existing control charts under Exponential distribution

| Shift	EWMA $V_1 = 3.689$		MEWMA $V_2 = 2.691$		TCC $V_3 = 4.125$		MEME $V_4 = 2.704$		MMEE $V_5 = 2.529$		MEME-TCC $V_6 = 4.396$		MMEE-TCC $V_7 = 3.808$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
0	370.27	259	370.53	259	370.59	256	370.21	258	370.30	258	370.68	256	370.70	257
0.05	253.09	177	252.04	176	311.36	217	219.45	165	223.01	166	**117.06**	**112**	209.07	164
0.1	180.79	127	178.76	126	239.09	166	143.68	126	143.76	127	**79.24**	**70**	137.81	126
0.25	80.69	58	80.25	57	123.54	86	53.17	57	53.74	57	**33.31**	**30**	53.09	56
0.50	33.76	25	33.38	25	54.76	38	20.59	25	20.62	26	**14.54**	**15**	20.49	24
0.75	19.34	15	19.34	15	30.37	21	12.29	14	12.32	15	**9.23**	**9**	12.06	14
1.00	13.12	10	13.29	10	19.41	13	9.00	9	9.02	10	**7.11**	**7**	9.08	9
1.50	7.83	6	8.09	7	10.17	7	6.28	5	6.29	5	**5.21**	4	6.21	4
2.00	5.53	5	5.85	5	6.45	4	5.14	5	5.15	5	**4.40**	4	5.06	4
3.00	**3.48**	3	3.54	3	3.51	2	4.05	4	4.08	4	3.59	3	4.05	3
4.00	**2.54**	**2**	2.92	2	2.94	1	3.48	4	3.49	4	3.15	3	3.47	3

Note: The fewest numbers of ARL_1 and MRL appear in bold.

Table 4. The ARL and MRL evaluations of the proposed chart and existing control charts under Gamma distribution

Shift	EWMA $V_1 = 1.887$		MEWMA $V_2 = 1.382$		TCC $V_3 = 2.591$		MEME $V_4 = 2.647$		MMEE $V_5 = 2.476$		MEME-TCC $V_6 = 3.356$		MMEE-TCC $V_7 = 3.107$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
0	370.33	258	370.51	257	370.12	255	370.20	257	370.33	258	370.55	256	370.56	257
0.05	285.33	174	284.59	197	334.85	231	244.69	171	247.95	172	**130.26**	**129**	219.95	170
0.1	230.65	126	229.15	152	295.44	206	213.57	125	214.32	126	**102.83**	**100**	211.45	125
0.25	116.07	57	115.95	75	204.60	142	93.54	55	93.84	56	**70.34**	**53**	93.09	55
0.50	49.86	26	49.66	28	115.79	80	40.87	24	41.88	25	**30.11**	**23**	40.39	24
0.75	29.54	13	29.52	14	69.01	48	20.92	12	21.65	13	**14.94**	**11**	20.16	13
1.00	14.89	11	**10.65**	8	43.19	30	14.18	10	14.74	10	11.01	8	14.04	9
1.50	8.13	8	**4.5**	4	18.86	13	8.05	7	8.11	7	5.01	4	8.10	6
2.00	5.79	6	**2.86**	**3**	9.30	6	6.62	5	6.64	5	3.03	4	6.61	5
3.00	2.57	2	**1.92**	2	3.79	2	4.07	4	4.09	4	2.61	3	4.08	3
4.00	1.44	1	**1.39**	1	2.98	1	3.46	2	3.48	2	1.96	2	3.47	2

Note: The fewest numbers of ARL_1 and MRL appear in bold.

Figures 2 to 5 show that the suggested chart's ARL curve remains on the bottom part for a wide range of small to moderate shifts, demonstrating the superiority of the proposed mixed control chart across all distributions. Even so, when the MRL finding was viewed, it was consistent with all ARL₁ distributions. Furthermore, the overall performance measures shown in Table 5 make it easier to understand this conclusion. As their overall performance measures from AEQL, PCL, and RMI values indicate, the proposed chart significantly outperformed across the full range of shifts in all distributions.
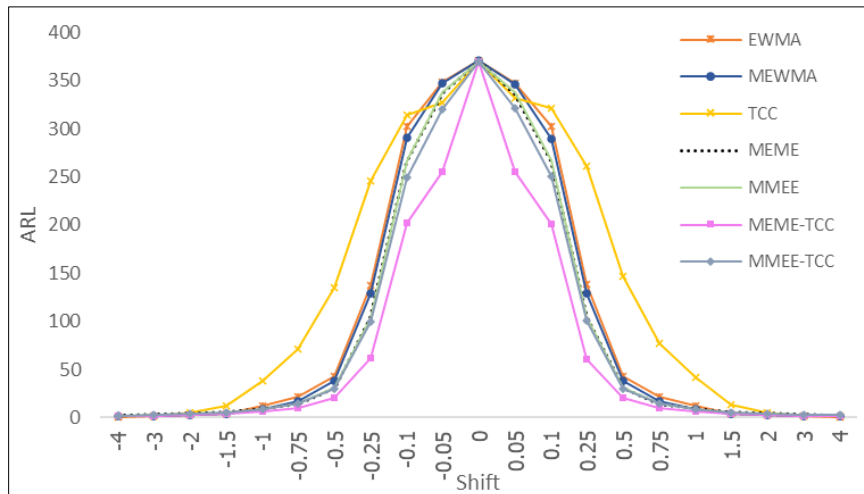
Figure 2. ARL-curve comparison of the proposed chart and existing control charts under the normal distribution
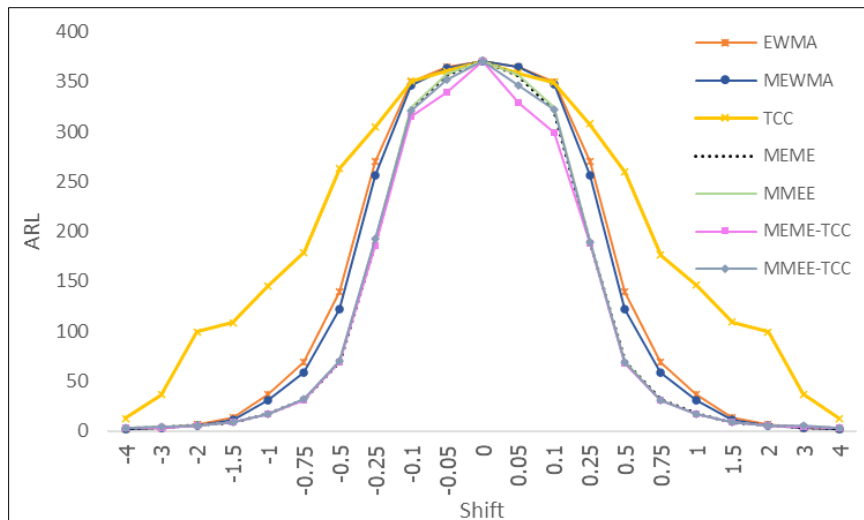
Figure 3. ARL-curve comparison of the proposed chart and existing control charts under Laplace distribution
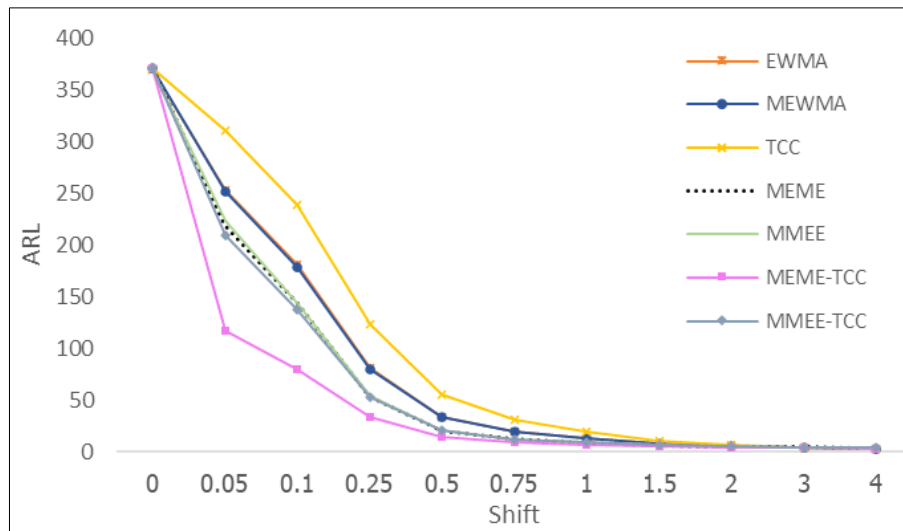
Figure 4. ARL-curve comparison of the proposed chart and existing control charts under Exponential distribution
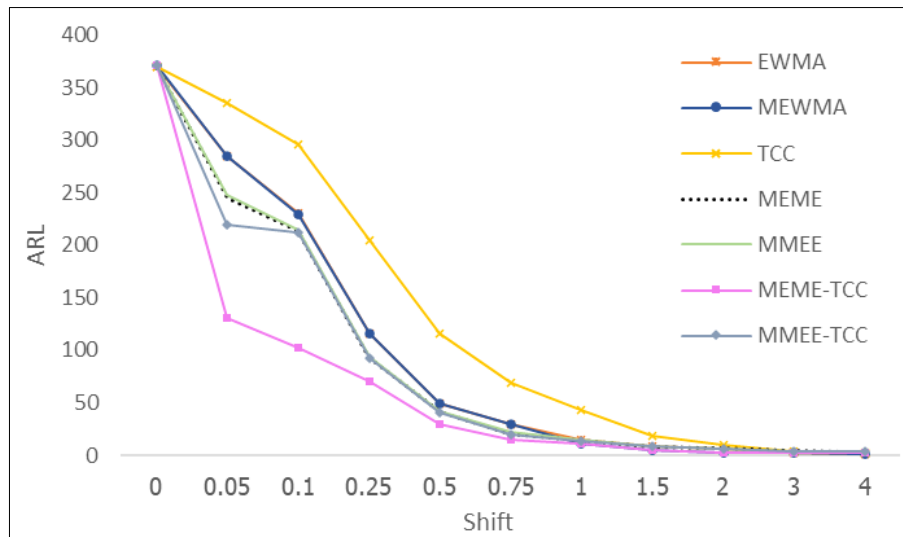

Figure 5. ARL-curve comparison of the proposed chart and existing control charts under Gamma distribution

Table 5. Comparison of control charts for overall performance

Distribution	Charts	Optimization criteria		
		AEQL	PCI	RMI
Normal	EWMA	8.02	1.01	0.49
	MEWMA	8.24	1.03	0.45
	TCC	20.85	2.61	2.52
	MEME	11.02	1.38	0.65
	MMEE	11.02	1.38	0.65
	MEME-TCC	**8.00**	**1.00**	**0.21**
	MMEE-TCC	11.02	1.38	0.64
Laplace	EWMA	21.78	1.25	0.44
	MEWMA	20.01	1.15	0.33
	TCC	137.76	7.89	5.71
	MEME	17.66	1.01	0.13
	MMEE	17.68	1.01	0.14
	MEME-TCC	**17.46**	**1.00**	**0.11**
	MMEE-TCC	19.25	1.10	0.19

	EWMA	13.78	1.16	0.72
	MEWMA	14.80	1.24	0.75
	TCC	16.25	1.36	1.34
Exponential	MEME	13.93	1.17	0.38
	MMEE	13.98	1.17	0.39
	MEME-TCC	**11.92**	**1.00**	**0.02**
	MMEE-TCC	13.85	1.16	0.36
	EWMA	12.90	1.28	0.67
	MEWMA	10.09	1.01	0.43
	TCC	26.27	2.61	2.04
Gamma	MEME	16.49	1.64	0.74
	MMEE	16.66	1.65	0.75
	MEME-TCC	**10.08**	**1.00**	**0.09**
	MMEE-TCC	16.44	1.63	0.71

Note: The fewest numbers appear in bold.

5- Illustrative Examples

Two illustrative examples are shown in this section. The first data set collected is 60 measurements of suspended solids from a specific lake [25]. The second data set was obtained on the lives of 36 specific types of batteries in an industrial setting [26]. We examined the data distribution and discovered that it followed the normal and gamma distributions, respectively. The data was used to generate the EWMA, MEWMA, MEME, MMEE, MMEE-TCC, and proposed (MEME-TCC) control charts.

The results of the first data sets in Figure 6 reveal that the suggested chart detects out-of-control signals at two sample positions, while the MEME chart detects them at three, the MEWMA, MMEE, and MMEE-TCC charts at four, and the EWMA chart at ten. The second data set in Figure 7 shows that the proposed chart detects out-of-control signals at one sample position, while the MEME chart detects them at three, the MEWMA, MMEE, and MMEE-TCC charts at five, and the EWMA chart at six. As a result, the MEME-TCC proposed chart detects shifts faster than the existing charts.
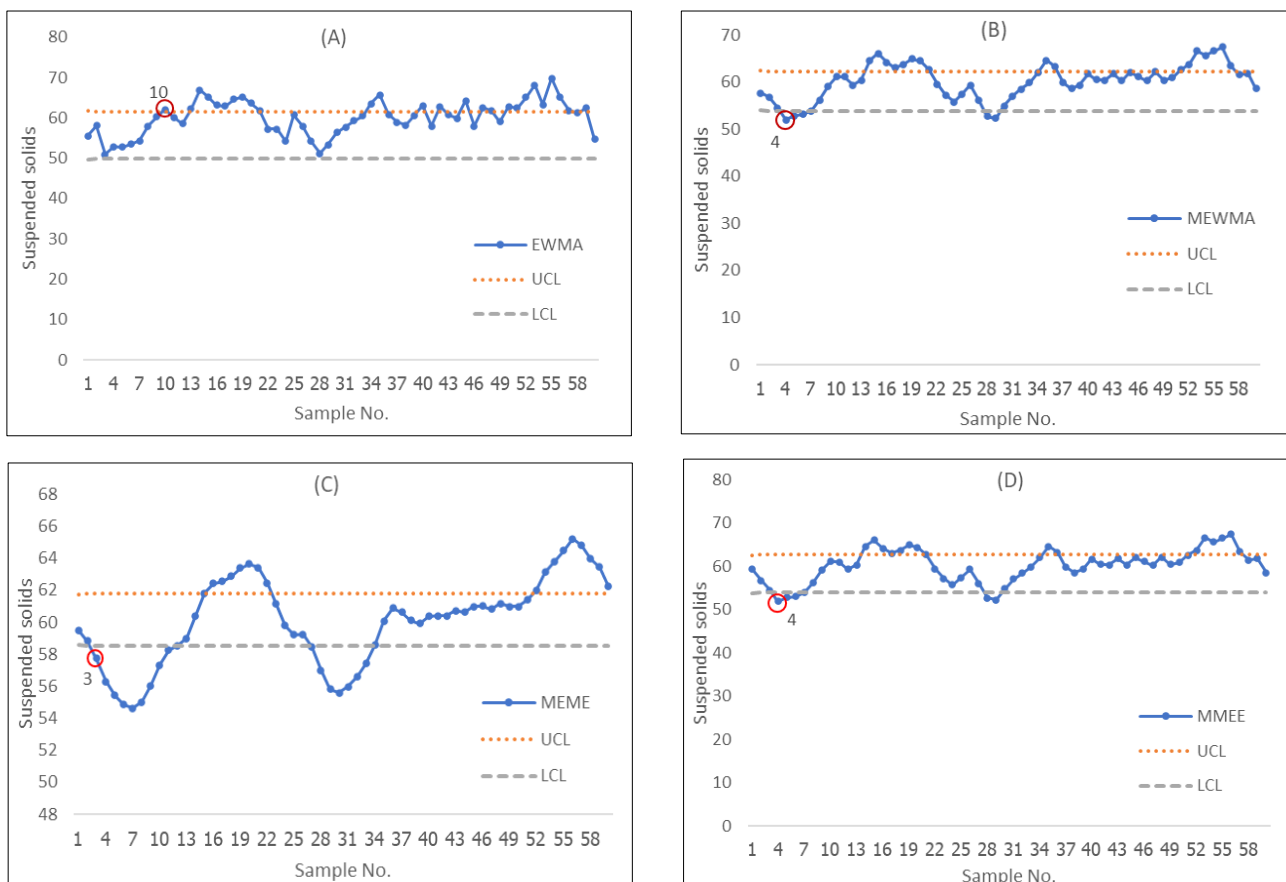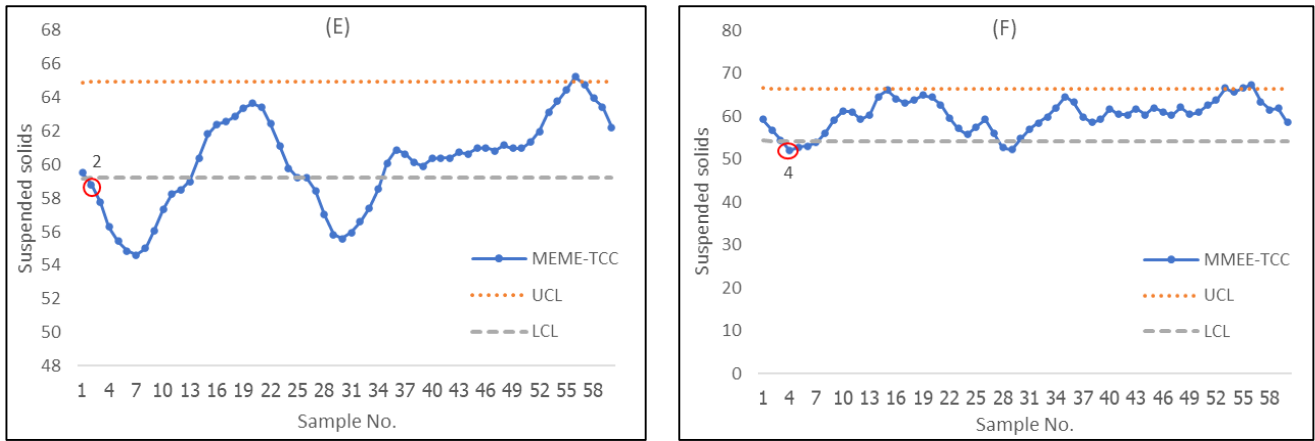
Figure 6. The effectiveness of the proposed chart and existing control charts for suspended solids data sets
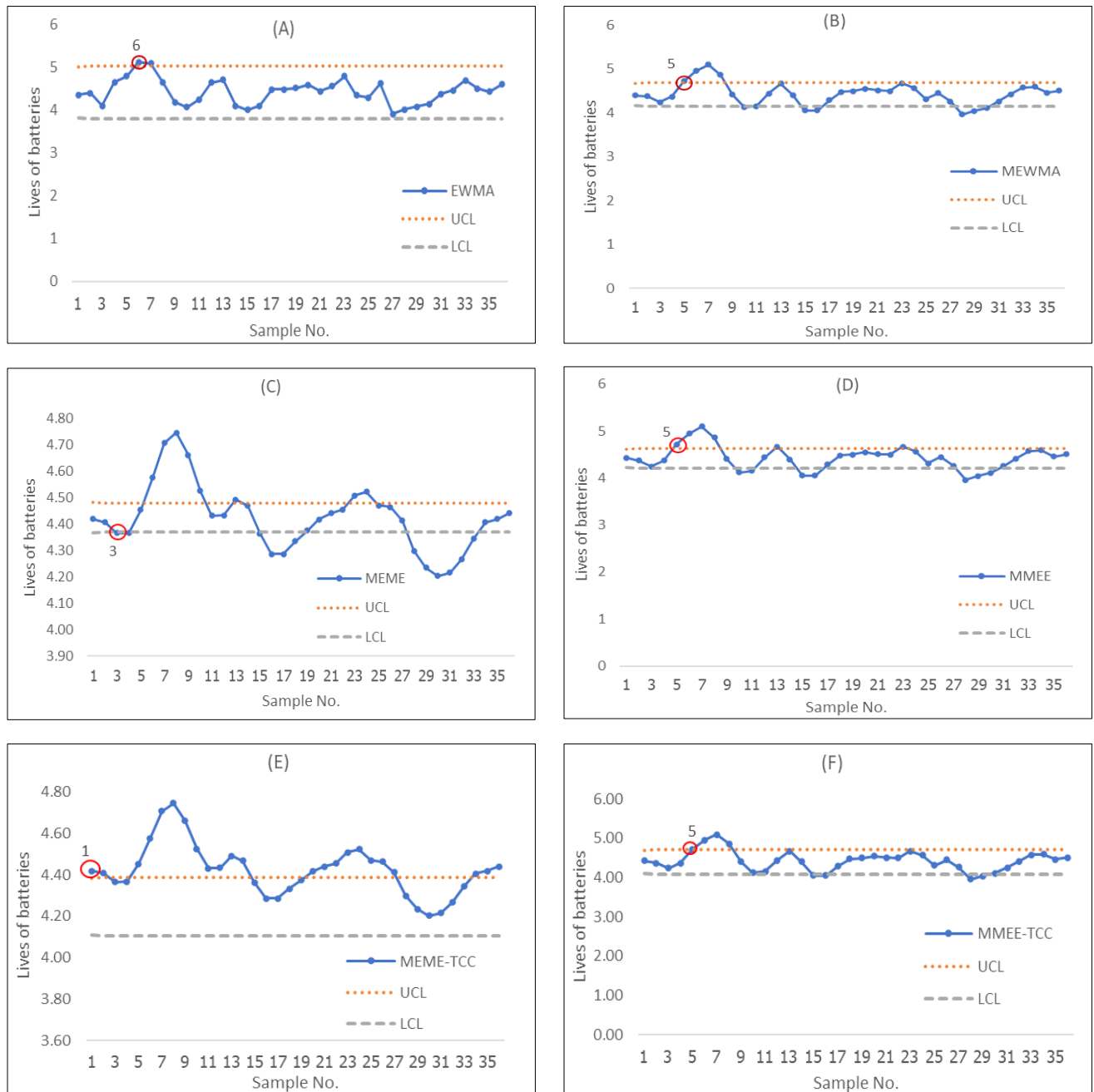

Figure 7. The effectiveness of the proposed chart and existing control charts for lives of batteries data sets

6- Conclusion

In this paper, we present a mixed control chart that combines the MEWMA with the EWMA chart and the nonparametric TCC to create better process mean monitoring schemes for symmetric and skewed distributions by using the ARL and MRL. The findings reveal that the proposed chart is really the best control chart, with the lowest ARL1 for small to moderate shifts among all distributional settings. Nevertheless, the EWMA chart detects large shifts more effectively than other charts, except in the case of the gamma distribution, where MEWMA performs best. Moreover, overall performance criteria derived from AEQL, PCL, and RMI values show that the proposed chart outperformed across the entire range of shifts in all distributions. A performance comparison in two data applications revealed that the suggested chart was capable of detecting shifts including both sets of data quickly. Besides that, we evaluate the ARL performance of the proposed chart to Tukey EWMA-MA [20] and Tukey EWMA-CUSUM [19] under normal distribution and ARL0 = 370 with shifts ranging from -4.00 to 4.00. The results of the simulation demonstrated that the proposed chart outperformed the Tukey EWMA - MA and Tukey EWMA-CUSUM charts in all shift dimensions. Even though, one of the study's limitations is that it takes a long time to simulate. The proposed chart may be chosen by quality practitioners as an efficient control chart for the non-normal process. The above work could be expanded in future studies to monitor variation in the process and apply it to real data with different distributions.

7- Declarations

7-1- Author Contributions

Conceptualization, S.S. and Y.A.; methodology, K.T. and S.S.; software, K.T.; validation, K.T., S.S. and Y.A.; formal analysis, S.S.; investigation, K.T. and Y.A.; resources, K.T.; data curation, K.T.; writing—original draft preparation, K.T.; writing—review and editing, S.S.; visualization, K.T. and Y.A.; supervision, S.S.; project administration, S.S.; funding acquisition, S.S. All authors have read and agreed to the published version of the manuscript.

7-2- Data Availability Statement

The data presented in this study are openly available in: Montgomery et al. [25] collected the dataset of suspended solids from a specific lake, and DeCoursey [26] collected the specific type battery dataset.

7-3- Funding

7-4- Acknowledgements

7-5- Institutional Review Board Statement

Not applicable.

7-6- Informed Consent Statement

Not applicable.

7-7- Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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