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Cannibalism and Harvesting in Tritrophic Chains: Insights from Mathematical and Artificial Neural Network Analysis

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Abstract

In this study, we introduce a novel tri-trophic food chain model that integrates cannibalism among basal prey and harvesting behaviors in the top predator, aiming to understand ecosystem dynamics comprehensively. Objectives encompass assessing system boundedness, computing fixed points, and determining stability characteristics using mathematical frameworks. The Routh-Hurwitz criteria and Lyapunov function are employed for local and global stability analyses of coexistence equilibrium points. Graphical interpretations elucidate relationships among pivotal parameters: prey growth rate, cannibalism intensity, and predator predation rate. Phase portraits and time series solutions illustrate parameter impacts. To enhance analytical depth and predictive capabilities, we utilize artificial neural networks (ANNs). Methods include connecting ANNs to computational proficiency for insights into the model's behavior over time. Findings demonstrate system boundedness, computed fixed points, and stability characteristics. Graphical interpretations reveal parameter impacts on system dynamics. ANNs offer predictive insights into a tri-trophic food chain model, employing mathematical analyses and ANNs to understand ecosystem dynamics comprehensively. Improvements include predictive capabilities and deeper analytical insights.

Keywords:

Food Chain; Cannibalism; Routh Hurwitz Criterion; Lyapunov Function; Global Stability; Artificial Neural Network.

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1- Introduction

Lotka (1925) [1] and Volterra (1926) [2] presented the first-ever models for competition phenomenon communication between prey and predators. Holling restructured these models by introducing three types of functional responses in prey-predator models to explain predation dynamics. Non-linear systems construct biological models of interest. That system is concerned with ordinary and partial differential equations. Prey-predator dynamics is a mathematical model generated from the cohabitation of foxes with rabbits. Foxes consume rabbits for their prey, which is a culture of the clover plant. As the number of foxes decreases in rabbits' habitat area, the rabbits' safety increases, and vice versa, i.e., if the number of rabbits decreases, the population of foxes increases [3]. Two species of animal populations are decreasing, and the other is growing. In a non-linear prey-predator system, population dynamics are represented in ordinary differential equations.

Mathematic ecology, which is of global importance, uses ordinary differential equations (ODEs). The existing and accurate models of ordinary differential equations (ODEs) are used to model biological systems [4]. The ODE models of biological systems are inherently non-linear. The first and second integrals are often difficult to come by, and this

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inherent difficulty makes it challenging to solve non-linear ordinary differential equations (ODEs) often used to model predators and prey.

These items are commonly classified as stiff or unrealistic and are known for posing significant challenges and complexities when attempting to discover analytical solutions. Hence, the effective and potent method for locating numerical solutions to complex non-linear systems has garnered significant interest. The research community's primary emphasis has been studying prey-predator models that incorporate interactions between two variables, namely interspecific interactions.

Nevertheless, Danca et al. [5], Jing & Yang [6], Liu and Xiao [7], and Elsadany et al. [8] have researched discretetime non-linear models that describe the dynamics of prey-predator interactions. The lifetime of dynamics in discrete time models exceeds that of continuous-time systems in terms of endurance. Summers et al. studied four ecosystem models affected by periodic forcing effects in discrete time [9]. Danca et al. conducted research with Holling to investigate the chaotic dynamics of discrete-time prey-predator systems [5]. Several innovative methods have recently been created to tackle these non-linear problems efficiently. The study employed various methods, such as the Runge-Kutta-Fehlberg method, the Laplace Adomian decomposition method [10], the differential transformation method [11], the finite element method [12], the Sumudu decomposition method [13], the Homotopy analysis method [14], and the new coupled fractional reduced differential transform method [15].

Food chains have a crucial role in the environment across several disciplines, including ecological science, applied mathematics, engineering, and economics. In a food chain model, organisms, energy, and resources flow along a single path. Food webs are complex due to their interconnection with multiple food chains. Various trophic levels have been seen in the food chain. Various categories of organisms in different stages include producers, consumers, and decomposers. Food chains play a vital role in the environment in various fields, such as ecological science, applied mathematics, engineering, and economics. Organisms, energy, and resources move linearly in a food chain model. Food webs are complex due to their interconnection with multiple food chains. Various trophic levels are present in the food chain. Various categories of organisms in different stages include producers, consumers, and decomposers. A food web employs a lattice architecture during its formation [16]. We may represent the food chain as a differential equation by employing mathematical analysis and modeling approaches. In ecology, food chains consist of a sequence of species that serve as a food source for the organisms immediately next to them. When many food chains are interconnected, they form a complex network known as a food web [17, 18]. The flexible food chain theory elucidates the structure and functioning of food webs with low trophic levels to understand the construction and dynamics of ecosystem stability [19, 20]. The life cycle of many species in nature is categorized into at least two groups, adult and juvenile, based on their behavior. The papers comprehensively examine food web models [21, 22]. The impact of cannibalism on the environmental perspective has been extensively discussed. Cannibalistic populations exist in terrestrial and marine food webs [23, 24]. Stage-structured individuals frequently engage in cannibalism, both within their population and throughout the aquatic food chain [25]. The article [26] examines the global stability of a predator-prey model, including both diffusive and non-diffusive dynamics. The model assumes that both species share a similar food source.

The authors in Ejaz et al. [27] and Arif et al. [28] have introduced a three-species predator-prey system where a predator and a prey species share a similar food source and illness in the interacting species. A fractional-order predatorprey model is introduced and examined in Arif et al. [29]. Artificial intelligence algorithms have effectively used stochastic computing paradigms to solve linear and non-linear models in many applications throughout applied science and technology [30, 31]. In Kumar et al. [32], authors look at a system of two fractional-order differential equations that describe the fear effect in prey-predator interactions, where the density of predators determines the rate of prey death. In Mollah & Sarwardi [33], they see the proposal of a three-dimensional prey-predator model that incorporates the factors of disease in predators and a temporal delay brought about by the gestation of the predator population. This work [34] examines the Lotka-Volterra model, a well-known dynamical system in mathematical biology, in its fractional version. For partial functional differential equations with non-linear diffusion, Xing & Jiang [35] established the exact formulae for the normal form coefficients associated with Turing-Hopf bifurcation. Recently, some researchers investigated the broader dynamics of tri-trophic food chain systems. In Kumari and Kumar [36], the authors investigated the impact of cannibalism within the middle predator. A range of cannibalism rates are used to investigate the impact of cannibalism on the stability property of the system. Different rates of cannibalism parameters are taken in the study, ranging from 0.02 to 0.29. In Kumari & Kumar [36], it is also shown that diffusion does not reveal a noticeable impact on the dynamic of the system. In Kar et al. [37], the authors investigated the tri-trophic food chain, and the Lyapunov function was constructed to show the global stability of the model. The author showed that under certain conditions the system shows stable behavior for the interior equilibrium point. It is observed that when basal prey population density is high, competition for resources for basal prey can increase, which may lead to cannibalism as an alternative food source. In the present study, we have shown that cannibalism in the basal prey impacts the interacting species' behavior. It is shown that the system is globally stable under some conditions. Moreover, a high cannibalism rate moves the system towards the stability of interacting populations for their coexistence. Below is an analysis of the identified gaps in the existing literature and the proposed approach to address them:

Identified Gaps in the Literature:

Limited Integration of Cannibalism and Human Harvesting Practices: Cannibalism among basic prey and the effect of human harvesting on top predators are complex dynamics that current ecological models frequently ignore. Unfortunately, existing models do not take cannibalism and harvesting into account nearly enough, despite the fact that these practices can drastically change population dynamics and ecological stability.

Challenges in Addressing Non-linearities and Stiff Dynamics: The non-linear dynamics present in ecological systems are difficult for traditional models grounded in ordinary differential equations (ODEs) to capture. The interactions between different species in food webs might be difficult to depict precisely due to rigid dynamics.

Insufficient Utilization of Innovative Methodologies: Diverse numerical methods and discrete-time non-linear models have been investigated in the past, but there has been little progress in applying these and other novel approaches to the complicated problems of ecological dynamics. A lot of the current models can only account for certain kinds of ecological interactions since they use old-fashioned mathematical methods.

Proposed Approaches to Fill the Gap:

Integration of Cannibalism and Harvesting in Ecological Models: The suggested method involves developing a tritrophic food chain model that explicitly includes cannibalism among the lowest-level prey and the impacts of human harvesting on the highest-level predators. Researchers will be able to assess the combined impact of these factors on population dynamics and stability with the help of an all-encompassing model, which will provide an improved comprehension of ecosystem dynamics.

Integration of Artificial Intelligence Algorithms: Using AI algorithms like ANNs and machine learning algorithms can enhance the analytical depth and predictive power of ecological models significantly. Artificial neural networks are ideally suitable for simulating the intricate dynamics of ecological systems as they are capable of identifying the intricate patterns and correlations within the data sets, and contributing AI algorithms to ecological models increases the accuracy and precision of these ecological models. Thus, these models can make sharper predictions and provide better comprehension regarding the general behavior of complex ecosystems.

The study adheres to a well-organized format with well-defined sections. In Section 2, a three-species tri-trophic ecological model is presented, including a description of its characteristics and an analysis of how cannibalism affects the basal prey. Section 3 presents the model's analysis, identifying equilibrium points, discussing their existence conditions, and exploring local and global stability through the Routh-Hurwitz criterion and a Volterra-type Lyapunov function. Section 4 discusses the outcomes of the results and simulation. Section 5 thoroughly analyzes the discussion. Section 6 offers a brief conclusion summarizing important discussions and suggests potential areas for future investigation.



Figure 1 shows the flowchart of the research methodology through which the objectives of this study were achieved.

Figure 1. Flow chart of the methodology

2- The Model

Ordinary differential equations (ODEs) proficiently explain the sophisticated dynamics of the interrelationships of species. These equations are a strong basis for understanding the complex behaviors in ecological systems. A combination of numerical methods and theoretical analysis is used to understand such a complex model. We will now

focus on a tri-trophic model that includes three species -a basal prey, a medium predator, and an apex predator. In this story, the main prey is a primary one. It has to swim through the complex environment of predation threats by both intermediate and apex predators. Thus, this model focuses on not only the external predation pressures but also the internal dynamics of the basic prey population. The internal dynamics involve the impact of cannibalism inside its community.

To quantify the complex set of interactions and dynamics of the ecosystem, it is important to establish its ecosystem through quantitative mathematical representations. These representations help establish the mathematical models. These interactions can be broken down into a series of differential equations. These equations depict the changes over time to intrinsic growth rates, predation rates, carrying capacities, and intra-specific interactions. It involves modeling a system of differential equations for the prey, the medium predator, and the apex predator. Population over time.

Interactions among the species drive the ecosystem dynamics in such a tri-trophic system. Every species plays a unique role in this model, made clear by the limitations and limitations of each species. The prey species that is basic has constraints on population expansion due to resources and predation and is the principal food source of the medium and top predators. The medium predator in this study maintains a balanced prey population because it is the prey of the top predator, and its principal food is the prey population. The top predator uses the medium predator as prey to build up pressure on the prey species, which is the basic prey population's food, to limit their population growth in this amicable system.

The model got more complex by introducing cannibalistic traits into the basal prey. This greatly influences the population dynamics of the basal prey by adding another rate of mortality that depends upon the abundance. It's another way to make the population interact with the inhabitants.

Mathematical representation and simulation with theoretical analysis help to gain insight into the tri-tropic system's stability, robustness, and behavior. This mathematical framework effectively helps us understand the complex relationship between species and environmental factors in ecological systems, which is crucial for better managing and preserving natural ecosystems.

$$\frac{dU}{dt} = U\left(1 - \frac{U}{k_1}\right)r_1 - UV\alpha_1 - UW\beta_1 - cU^2\tag{1}$$

$$\frac{dV}{dt} = V\left(1 - \frac{V}{k_2}\right)r_2 + UV\alpha_2 - VW\gamma_1 \tag{2}$$

$$\frac{dW}{dt} = UW\beta_2 + VW\gamma_2 - (h+d)W.$$
(3)

The parameters and their physical meanings are presented next.

Table 1 represents all the parameters and their physical meanings. The values of all parameters are taken to be positive. Moreover, U(0) > 0, V(0) > 0 and W(0) > 0. For the sake of simplicity, we take $h + d = \mu$.

Quantity	Physical Meanings				
U	Density of basal prey				
V	Density of medium predator				
W	Density of top predator				
k_{1}, k_{2}	Carrying capacity of U and V				
r_{1}, r_{2}	Intrinsic growth rate of U and V				
$\alpha_1, \beta_1, \gamma_1$	Predation rate of U by V and W and predation rate of V by W				
$\alpha_2, \beta_2, \gamma_2$	Growth rate of V and W due to predation				
С	Cannibalism rate of basal prey				
h, d	Death rate of a top predator due to harvesting and natural reasons				
μ	Combined death rate of a top predator due to harvesting and natural reasons				

Table 1. Parameters and their Description

3- Analysis of the Model

In this section, we present boundedness, the equilibrium points of the model, and the analysis of the fixed points for the global stability of coexistence fixed point constructing the Lyapunov function.

3-1-Boundedness of the System

Theorem 1: All solutions of the system (1-3) exhibit bounded behavior.

Proof: Considering the dynamics of each equation in the system (1-3), we derive the following inequalities:

$$\frac{dU}{dt} \leq U\left(1 - \frac{U}{k_1}\right)r_1$$

$$\frac{dU}{dt} \leq \frac{U}{k_1}(k_1 - U)r_1$$

$$U(t) \leq r_1k_1 - 2\frac{U}{k_1}r_1$$

$$U(t) \leq r_1k_1$$
(4)

which proves the boundedness of Equation 1.

Now, Equation 2 gives the following inequalities.

$$\frac{dv}{dt} \leq V\left(1 - \frac{V}{k_2}\right)r_2 + UV\alpha_2$$

$$\frac{dV}{dt} \leq V\left(\frac{k_2 - V}{k_2}\right)r_2 + UV\alpha_2$$

$$\frac{dV}{dt} \leq \frac{V}{k_2}(k_2 - V)r_2 + UV\alpha_2$$

$$V(t) \leq r_2k_2 + U\alpha_2$$

$$V(t) \leq r_2k_2 + \alpha_2r_1k_1$$
(5)

Similarly, Equation 3 gives

$$\frac{dW}{dt} \leq UW\beta_2 + VW\gamma_2$$

$$W(t) \leq U(t)\beta_2 + V(t)\gamma_2$$

$$W(t) \leq (r_1k_1)\beta_2 + (r_2k_2 + \alpha_2r_1k_1)\gamma_2$$
(6)

Hence, the theorem.

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3-2-Equilibrium Points and Stability

We present equilibrium points of the model (1-3) and discuss the stability of the coexistence equilibrium point.

Consider the following system of equations for computing the equilibrium points of the model presented in (1-3)

$$-cU^{2} + U\left(1 - \frac{U}{k_{1}}\right)r_{1} - UV\alpha_{1} - UW\beta_{1} = 0,$$
(7)

$$V\left(1-\frac{V}{k_2}\right)r_2 + UV\alpha_2 - VW\gamma_1 = 0,\tag{8}$$

$$-W\mu + UW\beta_2 + VW\gamma_2 = 0. \tag{9}$$

The solution of the above system leads to the following equilibrium points.

$$\begin{split} \mathbf{E}_{1} &= \begin{pmatrix} 0, & \frac{\mu}{\gamma_{2}}, & \frac{r_{2}(-\mu+k_{2}\gamma_{2})}{k_{2}\gamma_{1}\gamma_{2}} \end{pmatrix}, \\ \mathbf{E}_{2} &= (0, & k_{2}, & 0), \\ \mathbf{E}_{3} &= \begin{pmatrix} \frac{k_{1}(r_{2}\beta_{1}(\mu-k_{2}\gamma_{2})+k_{2}\gamma_{1}(-\mu\alpha_{1}+r_{1}\gamma_{2}))}{k_{1}\beta_{2}(r_{2}\beta_{1}-k_{2}\alpha_{1}\gamma_{1})+k_{2}(k_{1}\alpha_{2}\beta_{1}+(ck_{1}+r_{1})\gamma_{1})\gamma_{2}}, \frac{k_{2}(\mu r_{1}\gamma_{1}+k_{1}(\beta_{1}(\mu\alpha_{2}+r_{2}\beta_{2})+(c\mu-r_{1}\beta_{2})\gamma_{1}))}{k_{1}\beta_{2}(r_{2}\beta_{1}-k_{2}\alpha_{1}\gamma_{1})+k_{2}(k_{1}\alpha_{2}\beta_{1}+(ck_{1}+r_{1})\gamma_{1})\gamma_{2}}, \\ & \frac{r_{1}r_{2}(-\mu+k_{2}\gamma_{2})+k_{1}(r_{2}(-c\mu+(r_{1}-k_{2}\alpha_{1})\beta_{2}+ck_{2}\gamma_{2})+k_{2}\alpha_{2}(-\mu\alpha_{1}+r_{1})\gamma_{1})\gamma_{2}}{k_{1}\beta_{2}(r_{2}\beta_{1}-k_{2}\alpha_{1}\gamma_{1})+k_{2}(k_{1}\alpha_{2}\beta_{1}+(ck_{1}+r_{1})\gamma_{1})\gamma_{2}}, \end{pmatrix} \\ & \mathbf{E}_{4} &= \begin{pmatrix} \frac{\mu}{\beta_{2}}, & 0, & -\frac{c\mu k_{1}+\mu r_{1}-k_{1}r_{1}\beta_{2}}{k_{1}\beta_{1}\beta_{2}} \end{pmatrix}, \end{split}$$

$$\begin{split} \mathbf{E}_{5} &= \left(\frac{k_{1}r_{1}}{ck_{1}+r_{1}}, \quad 0, \quad 0\right), \\ \mathbf{E}_{6} &= \left(-\frac{-k_{1}r_{1}r_{2}+k_{1}k_{2}r_{2}\alpha_{1}}{ck_{1}r_{2}+r_{1}r_{2}+k_{1}k_{2}\alpha_{1}\alpha_{2}}, \quad \frac{k_{2}(ck_{1}r_{2}+r_{1}r_{2}+k_{1}r_{1}\alpha_{2})}{ck_{1}r_{2}+r_{1}r_{2}+k_{1}k_{2}\alpha_{1}\alpha_{2}}, \quad 0\right), \\ \mathbf{E}_{7} &= (0, \quad 0, \quad 0) \end{split}$$

Theorem 2: The Equations 1 to 3 are locally stable for the coexistence equilibrium point E_3 if following holds

- i. $m_{11} + m_{22} < 0$,
- ii. $m_{13}m_{22}m_{31} + m_{11}m_{23}m_{32} > m_{12}m_{23}m_{31} + m_{13}m_{21}m_{32}$,
- iii. $m_{11}m_{22} > m_{12}m_{21} + m_{13}m_{31} + m_{23}m_{32}$,

iv.
$$m_{11}(-m_{12}m_{21} + m_{22}(m_{11} + m_{22}) - m_{13}m_{31}) < m_{12}(m_{21}m_{22} + m_{23}m_{31}) + (m_{13}m_{21} + m_{22}m_{23})m_{32}$$

where,

$$\begin{split} m_{11} &= -\frac{(ck_1 + r_1)(r_2\beta_1(\mu - k_2\gamma_2) + k_2\gamma_1(-\mu\alpha_1 + r_1\gamma_2))}{k_1\beta_2(r_2\beta_1 - k_2\alpha_1\gamma_1) + k_2(k_1\alpha_2\beta_1 + (ck_1 + r_1)\gamma_1)\gamma_2'}, \\ m_{12} &= -\frac{k_1\alpha_1(r_2\beta_1(\mu - k_2\gamma_2) + k_2\gamma_1(-\mu\alpha_1 + r_1\gamma_2))}{k_1\beta_2(r_2\beta_1 - k_2\alpha_1\gamma_1) + k_2(k_1\alpha_2\beta_1 + (ck_1 + r_1)\gamma_1)\gamma_2'}, \\ m_{13} &= \frac{k_1\beta_1(r_2\beta_1(-\mu + k_2\gamma_2) + k_2\gamma_1(\mu\alpha_1 - r_1\gamma_2))}{k_1\beta_2(r_2\beta_1 - k_2\alpha_1\gamma_1) + k_2(k_1\alpha_2\beta_1 + (ck_1 + r_1)\gamma_1)\gamma_2'}, \\ m_{21} &= \frac{k_2\alpha_2(\mu r_1\gamma_1 + k_1(\beta_1(\mu\alpha_2 + r_2\beta_2) + (c\mu - r_1\beta_2)\gamma_1)))}{k_1\beta_2(r_2\beta_1 - k_2\alpha_1\gamma_1) + k_2(k_1\alpha_2\beta_1 + (ck_1 + r_1)\gamma_1)\gamma_2'}, \\ m_{22} &= \frac{r_2(-\mu r_1\gamma_1 - k_1(\beta_1(\mu\alpha_2 + r_2\beta_2) + (c\mu - r_1\beta_2)\gamma_1))}{k_1\beta_2(r_2\beta_1 - k_2\alpha_1\gamma_1) + k_2(k_1\alpha_2\beta_1 + (ck_1 + r_1)\gamma_1)\gamma_2'}, \\ m_{31} &= \frac{\beta_2(r_1r_2(-\mu + k_2\gamma_2) + k_1(r_2(-c\mu + (r_1 - k_2\alpha_1)\beta_2 + ck_2\gamma_2) + k_2\alpha_2(-\mu\alpha_1 + r_1\gamma_2)))}{k_1\beta_2(r_2\beta_1 - k_2\alpha_1\gamma_1) + k_2(k_1\alpha_2\beta_1 + (ck_1 + r_1)\gamma_1)\gamma_2}, \\ m_{32} &= \frac{\gamma_2(r_1r_2(-\mu + k_2\gamma_2) + k_1(r_2(-c\mu + (r_1 - k_2\alpha_1)\beta_2 + ck_2\gamma_2) + k_2\alpha_2(-\mu\alpha_1 + r_1\gamma_2)))}{k_1\beta_2(r_2\beta_1 - k_2\alpha_1\gamma_1) + k_2(k_1\alpha_2\beta_1 + (ck_1 + r_1)\gamma_1)\gamma_2}. \end{split}$$

Proof. We find Jacobian for the system (1-3), which is given as under.

$$J = \begin{pmatrix} -2cU + (1 - \frac{U}{k_1})r_1 - \frac{Ur_1}{k_1} - V\alpha_1 - V\beta_1 & -U\alpha_1 & -U\beta_1 \\ V\alpha_2 & (1 - \frac{V}{k_2})r_2 - \frac{Vr_2}{k_2} + U\alpha_2 - V\gamma_1 & -V\gamma_1 \\ V\beta_2 & V\gamma_2 & -\mu + U\beta_2 + V\gamma_2 \end{pmatrix}.$$
 (10)

Now we find Jacobian at the fixed point E_3

$$J_{E_3} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & 0 \end{bmatrix}$$
(11)

where,

$$\begin{split} m_{11} &= -\frac{(ck_1+r_1)(r_2\beta_1(\mu-k_2\gamma_2)+k_2\gamma_1(-\mu\alpha_1+r_1\gamma_2))}{k_1\beta_2(r_2\beta_1-k_2\alpha_1\gamma_1)+k_2(k_1\alpha_2\beta_1+(ck_1+r_1)\gamma_1)\gamma_2},\\ m_{12} &= -\frac{k_1\alpha_1(r_2\beta_1(\mu-k_2\gamma_2)+k_2\gamma_1(-\mu\alpha_1+r_1\gamma_2))}{k_1\beta_2(r_2\beta_1-k_2\alpha_1\gamma_1)+k_2(k_1\alpha_2\beta_1+(ck_1+r_1)\gamma_1)\gamma_2}, \end{split}$$

$$\begin{split} m_{13} &= \frac{k_1 \beta_1 (r_2 \beta_1 (-\mu + k_2 \gamma_2) + k_2 \gamma_1 (\mu \alpha_1 - r_1 \gamma_2))}{k_1 \beta_2 (r_2 \beta_1 - k_2 \alpha_1 \gamma_1) + k_2 (k_1 \alpha_2 \beta_1 + (ck_1 + r_1) \gamma_1) \gamma_2'}, \\ m_{21} &= \frac{k_2 \alpha_2 (\mu r_1 \gamma_1 + k_1 (\beta_1 (\mu \alpha_2 + r_2 \beta_2) + (c\mu - r_1 \beta_2) \gamma_1))}{k_1 \beta_2 (r_2 \beta_1 - k_2 \alpha_1 \gamma_1) + k_2 (k_1 \alpha_2 \beta_1 + (ck_1 + r_1) \gamma_1) \gamma_2'}, \\ m_{22} &= \frac{r_2 (-\mu r_1 \gamma_1 - k_1 (\beta_1 (\mu \alpha_2 + r_2 \beta_2) + (c\mu - r_1 \beta_2) \gamma_1))}{k_1 \beta_2 (r_2 \beta_1 - k_2 \alpha_1 \gamma_1) + k_2 (k_1 \alpha_2 \beta_1 + (ck_1 + r_1) \gamma_1) \gamma_2'}, \\ m_{23} &= -\frac{k_2 \gamma_1 (\mu r_1 \gamma_1 + k_1 (\beta_1 (\mu \alpha_2 + r_2 \beta_2) + (c\mu - r_1 \beta_2) \gamma_1))}{k_1 \beta_2 (r_2 \beta_1 - k_2 \alpha_1 \gamma_1) + k_2 (k_1 \alpha_2 \beta_1 + (ck_1 + r_1) \gamma_1) \gamma_2'}, \\ m_{31} &= \frac{\beta_2 (r_1 r_2 (-\mu + k_2 \gamma_2) + k_1 (r_2 (-c\mu + (r_1 - k_2 \alpha_1) \beta_2 + ck_2 \gamma_2) + k_2 \alpha_2 (-\mu \alpha_1 + r_1 \gamma_2)))}{k_1 \beta_2 (r_2 \beta_1 - k_2 \alpha_1 \gamma_1) + k_2 (k_1 \alpha_2 \beta_1 + (ck_1 + r_1) \gamma_1) \gamma_2}, \\ m_{32} &= \frac{\gamma_2 (r_1 r_2 (-\mu + k_2 \gamma_2) + k_1 (r_2 (-c\mu + (r_1 - k_2 \alpha_1) \beta_2 + ck_2 \gamma_2) + k_2 \alpha_2 (-\mu \alpha_1 + r_1 \gamma_2)))}{k_1 \beta_2 (r_2 \beta_1 - k_2 \alpha_1 \gamma_1) + k_2 (k_1 \alpha_2 \beta_1 + (ck_1 + r_1) \gamma_1) \gamma_2} \end{split}$$

The characteristic polynomial can be written as

$$P(\lambda) = \lambda^{3} + \lambda^{2}(-m_{11} - m_{22}) + m_{13}m_{22}m_{31} - m_{12}m_{23}m_{31} - m_{13}m_{21}m_{32} + m_{11}m_{23}m_{32} + \lambda(-m_{12}m_{21} + m_{11}m_{22} - m_{13}m_{31} - m_{23}m_{32}).$$
(12)

The equilibrium point will be stable under the following conditions

$$m_{11} + m_{22} < 0 \tag{13}$$

$$m_{13}m_{22}m_{31} + m_{11}m_{23}m_{32} > m_{12}m_{23}m_{31} + m_{13}m_{21}m_{32}$$
⁽¹⁴⁾

$$m_{11}m_{22} > m_{12}m_{21} + m_{13}m_{31} + m_{23}m_{32} \tag{15}$$

and

$$m_{11}(-m_{12}m_{21} + m_{22}(m_{11} + m_{22}) - m_{13}m_{31}) < m_{12}(m_{21}m_{22} + m_{23}m_{31}) + (m_{13}m_{21} + m_{22}m_{23})m_{32}$$
(16)

Hence, the proof.

Theorem 3: The system (1-3) is globally asymptotically stable for the coexistence equilibrium point E_3 under the following conditions.

- i. $\alpha_1 > \alpha_2, \beta_1 > \beta_2$ and $\gamma_1 > \gamma_2$,
- ii. U, V, and W must deviate from U^* , V^* and W^* .

Proof: Consider the following Lyapunov function for the coexistence equilibrium point $E_3(U^*, V^*, W^*)$, where,

$$U^{*} = \frac{k_{1}(r_{2}\beta_{1}(\mu - k_{2}\gamma_{2}) + k_{2}\gamma_{1}(-\mu\alpha_{1} + r_{1}\gamma_{2}))}{k_{1}\beta_{2}(r_{2}\beta_{1} - k_{2}\alpha_{1}\gamma_{1}) + k_{2}(k_{1}\alpha_{2}\beta_{1} + (ck_{1} + r_{1})\gamma_{1})\gamma_{2}},$$

$$V^{*} = \frac{k_{2}(\mu r_{1}\gamma_{1} + k_{1}(\beta_{1}(\mu\alpha_{2} + r_{2}\beta_{2}) + (c\mu - r_{1}\beta_{2})\gamma_{1}))}{k_{1}\beta_{2}(r_{2}\beta_{1} - k_{2}\alpha_{1}\gamma_{1}) + k_{2}(k_{1}\alpha_{2}\beta_{1} + (ck_{1} + r_{1})\gamma_{1})\gamma_{2}},$$

$$W^{*} = \frac{r_{1}r_{2}(-\mu + k_{2}\gamma_{2}) + k_{1}(r_{2}(-c\mu + (r_{1} - k_{2}\alpha_{1})\beta_{2} + ck_{2}\gamma_{2}) + k_{2}\alpha_{2}(-\mu\alpha_{1} + r_{1}\gamma_{2}))}{k_{1}\beta_{2}(r_{2}\beta_{1} - k_{2}\alpha_{1}\gamma_{1}) + k_{2}(k_{1}\alpha_{2}\beta_{1} + (ck_{1} + r_{1})\gamma_{1})\gamma_{2}}.$$

$$L(U, V, W) = l_{1}\left(U - U^{*} - U^{*}log\frac{U}{U^{*}}\right) + l_{2}\left(V - V^{*} - V^{*}log\frac{V}{V^{*}}\right) + l_{3}\left(W - W^{*} - W^{*}log\frac{W}{W^{*}}\right)$$
(17)

where l_1 , l_2 and l_3 are positive constants whose value is to be determined.

The derivative of the above equation leads to the following result.

$$\frac{d\mathcal{L}}{dt} = l_1 \left(1 - \frac{\mathcal{U}^*}{\mathcal{U}} \right) \frac{d\mathcal{U}}{dt} + l_2 \left(1 - \frac{\mathcal{V}^*}{\mathcal{V}} \right) \frac{d\mathcal{V}}{dt} + l_3 \left(1 - \frac{\mathcal{W}^*}{\mathcal{W}} \right) \frac{d\mathcal{W}}{dt}.$$
(18)

Some simplification of the above leads to

$$\frac{dL}{dt} = l_1 \left(\frac{U-U^*}{U}\right) \frac{dU}{dt} + l_2 \left(\frac{V-V^*}{V}\right) \frac{dV}{dt} + l_3 \left(\frac{W-W^*}{W}\right) \frac{dW}{dt}.$$
(19)

$$\frac{dL}{dt} = l_1 (U - U^*) \left[-cU + (1 - \frac{U}{k_1})r_1 - V\alpha_1 - W\beta_1 \right] + l_2 (V - V^*) \left[(1 - \frac{V}{k_2})r_2 + U\alpha_2 - W\gamma_1 \right] + l_3 (W - W^*) \left[-\mu + U\beta_2 + V\gamma_2 \right]$$
(20)

As we have;

$$-cU^* + \left(1 - \frac{U^*}{k_1}\right)r_1 - V^*\alpha_1 - W^*\beta_1 = 0 \Rightarrow r_1 = \frac{r_1U^*}{k_1} + cU^* + V^*\alpha_1 + W^*\beta_1.$$
(21)

Also
$$\left(1 - \frac{V^*}{k_2}\right)r_2 + U^*\alpha_2 - W^*\gamma_1 = 0 \Rightarrow r_2 = \frac{r_2V^*}{K_2} - U^*\alpha_2 + W^*\gamma_1.$$
 (22)

And,

$$-\mu + U^* \beta_2 + V^* \gamma_2 = 0 \implies \mu = U^* \beta_2 + V^* \gamma_2.$$
⁽²³⁾

Using Equations 21 to 23 in Equation 20, we have

$$\frac{dL}{dt} = l_1(U - U^*) \left[-cU - \frac{U}{k_1} r_1 - V\alpha_1 - W\beta_1 + \frac{r_1 U^*}{k_1} + cU^* + V^*\alpha_1 + W^*\beta_1 \right] + l_2(V - V^*) \left[\frac{r_2 V^*}{K_2} - U^*\alpha_2 + W^*\gamma_1 - \frac{V}{k_2} r_2 + U\alpha_2 - W\gamma_1 \right] + l_3(W - W^*) [-U^*\beta_2 - V^*\gamma_2 + U\beta_2 + V\gamma_2].$$
(24)

$$\frac{dL}{dt} = l_1(U - U^*) \left[-c(U - U^*) - \frac{r_1}{k_1}(U - U^*) - \alpha_1(V - V^*) - \beta_1(W - W^*) \right] + l_2(V - V^*) \left[-\frac{r_2}{k_2}(V - V^*) + (U - U^*)\alpha_2 - \gamma_1(W - W^*) \right] + l_3(W - W^*) \left[(U - U^*)\beta_2 + (V - V^*)\gamma_2 \right]$$
(25)

For simplicity, we assume that;

$$l_1 = l_2 = l_3 = 1. (26)$$

Using Equation 26 in Equation 25, we get the following:

$$\frac{dL}{dt} = -(c + \frac{r_1}{k_1})(U - U^*)^2 - \alpha_1(U - U^*)(V - V^*) - \beta_1(W - W^*)(U - U^*) - \frac{r_2}{K_2}(V - V^*)^2 + \alpha_2(U - U^*)(V - V^*) - \gamma_1(W - W^*)(V - V^*) + \beta_2(W - W^*)(U - U^*) + \gamma_2(V - \dot{V}^*)(W - W^*).$$
(27)

Some simplification leads to

$$\frac{dL}{dt} = -(c + \frac{r_1}{k_1})(U - U^*)^2 - (\alpha_1 - \alpha_2)(U - U^*)(V - V^*) - (\beta_1 - \beta_2)(W - W^*)(U - U^*) - \frac{r_2}{K_2}(V - V^*)^2 - (\gamma_1 - \gamma_2)(W - W^*)(V - V^*).$$
(28)

The coexistence equilibrium point is stable iff the following hold.

- i. α₁ > α₂, β₁ > β₂ and γ₁ > γ₂,
 ii. U, V and W must deviate from U*, V* and W*

Now, $\frac{dL}{dt} < 0$ if the above conditions i and ii hold. Hence, the theorem is proved.

Figure 2 depicts a hidden circuit, a recurrent neural network (RNN), used to protect privacy or security. It consists of three layers of neurons, with the first layer taking input, the second layer taking output, and the third layer taking output. The weights represent connections, biases, and summation. The arrows indicate the direction of information flow through the hidden circuit, from input to output. This helps us understand the RNN's functionality.



Figure 2. Schematic representation of a recurrent neural network (RNN) including a concealed layer

Table 2 summarizes the simulation with varying cannibalism rate (c) and death rate of prey (μ). It includes performance metrics like Gradient, learning rate (Mu), epochs, and time for training, validation, and testing scenarios. The table provides a concise overview of the model's performance under different conditions, allowing a comparison of the effects of varying cannibalism and death rates on training speed, learning efficiency, and overall performance.

Table 2. Simulation Results: Predator-prey dynamics with varying cannibalism rate (c) and death rate of top predator (μ). Performance metrics include Gradient (Grad), learning rate (Mu), epochs, and time for training, validation, and testing scenarios.

Physical Quantities		Training	Validation	Testing	Performance	Grad	Mu	Epochs	Time
Cannibalism rate of basal prey (c)	0.03	1.129×10 ⁻¹⁰	2.19×10 ⁻¹⁰	8.09×10 ⁻¹⁰	3.07×10 ⁻⁹	1.29×10 ⁻⁵	1.07×10 ⁻⁷	312	11
	0.01	1.93×10 ⁻¹⁰	2.09×10 ⁻¹⁰	6.01×10 ⁻¹⁰	3.21×10 ⁻⁹	3.9×10 ⁻⁹	1.9×10-8	234	5
Death Rate of Top Predator (μ)	0.9	1.56×10-9	2.16×10-9	5.16×10 ⁻⁹	4.56×10 ⁻⁹	6.7×10 ⁻⁹	1.9×10-8	567	8
	0.10	1.46×10-9	2.96×10 ⁻⁹	5.96×10 ⁻⁹	3.6×10-9	6.07×10 ⁻⁹	1.8×10 ⁻⁸	560	7

4- Results and Discussion

Here, we present the simulation results of the model (1-3). The solution of the model is provided using ode45. Time series solutions and phase portraits are presented to show the impact of key parameters like intrinsic growth rate, cannibalism, and death rate of top-predator species.

Figure 3 depicts the impact of the intrinsic rate growth of basal prey on the interacting population for two different values of the said rate. Figures 3-a and 3-b clearly show that the parameter significantly affects the population. In these plots, the values of the parameters are taken as follows. $r_1 = 0.49978$, $r_2 = 0.50980$, $\alpha_1 = 67581$, $\beta_1 = 0.5600$, $\gamma_1 = 0.08794$, $\alpha_2 0.645$, $\beta_2 = 0.2549$, $\gamma_2 = 0.0643$, c = 0.3421, $\mu = 0.00854$ whereas the values of r_1 are taken as $r_1 = 0.49978$ and $r_1 = 0.99978$.

Figures 3-a and 3-b depict a simulated food chain where the growth rate of the basal prey significantly influences the entire system's stability. In the first figure, with a very high growth rate for U, we see a boom-and-bust cycle. The basal prey flourishes initially, leading to a surge in medium predators. However, this abundance attracts top predators, who then feast on the medium predator, causing their population to crash. This decline in medium predators eventually reduces the food source for the top predators, potentially leading to their population decline as well. The cycle can then repeat as the basal prey recovers. In contrast, the second figure shows a more stable system with a lower growth rate. The limited growth restricts the medium predator population, resulting in smaller fluctuations for both herbivores and top predators. This comparison highlights how the intrinsic growth rate of the basal prey plays a critical role in shaping the dynamics of the food chain. A higher growth rate can lead to dramatic fluctuations, while a lower growth rate promotes a more stable system.



(a) Time series and phase portrait at r=0.49978





Figure 3. Impact of r_1 on the population dynamics for two different intrinsic growth rate

Figure 4 depicts the impact of the cannibalism effect of prey on the interacting population for two different values of the said rate. Figures 4-a and 4-b clearly show that the parameter significantly affects the population. In these plots, the values of the parameters are taken the same as in Figure 3 for two different values of c=0.03421 and 0.013421, respectively, whereas as $r_1 = 0.49978$. It is evident from the plots that decreasing the parameter's value increases the oscillations in the solution, and the limit cycle is evident in Figure 4-b. Figure 5 depicts the impact of harvesting of top predators on the interacting population for two different values of the said rate. Figures 5-a and 5-b clearly show that the parameter significantly affects the population. In these plots, the values of the parameters are taken the same as in Figure 3 for two different values of μ =0.90085 and 0.10085, respectively, whereas as $r_1 = 0.49978$ and c=0.013421.

These two, Figures 4-a and 4-b, depict a simulated food chain where the basal prey can consume each other. Figure 4-a has a higher cannibalism rate (c = 0.03421) compared to Figure 4-b (c = 0.013421). The presence of cannibalism can potentially reduce the dramatic fluctuations sometimes seen in food chains. By consuming each other, the plants (basal prey) help regulate their population growth. This can lead to smaller fluctuations in the populations of medium and top predators compared to scenarios without cannibalism. By comparing these two plots, we can see that the lower cannibalism rate in Figure 4-b might be associated with slightly larger fluctuations in the populations compared to the higher cannibalism rate in Figure 4-a.

Moving to Figure 5, the focus shifts to the impact of harvesting the top predator on the interacting population. Figures 5-a and 5-b echo the trend observed in the previous plots, emphasizing the significant influence of this parameter on the population. The parameter values for this scenario align with those in Figure 3, while μ assumes two different values, 0.90085 and 0.10085, with $r_1 = 0.49978$ and c=0.013421. Analogous to Figure 4, a decrease in the parameter μ intensifies oscillations in the solution, culminating in an evident limit cycle as portrayed in Figure 5-b, underscoring the behavior identified in Figure 4.

The plots show how this death rate can affect the population dynamics of all three trophic levels. When the top predator death rate is high (Figure 5-a), the top predator population appears smaller. This potentially allows the medium predator population to grow with less predation pressure. However, the primary producers might experience increased grazing pressure from the larger top predator population. In contrast, a lower death rate for top predators (Figure 5-b) suggests a larger and more stable carnivore population. This can lead to increased predation on herbivores (green line), potentially regulating their population at a lower density. Consequently, the primary producers (blue line) might experience less grazing pressure and potentially show larger fluctuations or reach higher densities.



Figure 4. Impact of μ on the population dynamics for two different harvesting rates

Figure 6 disentangles the impact of the growth rate of the top predator stemming from predating the basal prey. This elegantly crafted graph illuminates the population's response to two distinct values of this pivotal rate. It can be observed from the multifaceted choreography depicted in Figures 6-a and 6-b that this parameter exerts a profound influence on the population. Remarkably, these plots maintain parameter consistency with the preceding narrative in Figure 5, holding steadfast to the values that have woven the complex ecological tale. The growth rate parameter μ stands at 0.10085, while the dynamic duo of β_2 assumes the distinct values of 0.02459 and 0.00124, crafting a visual narrative that articulates the sensitivity of population dynamics to this parameter. The plots gracefully illustrate a key observation: the reduction in the parameter value is met with an amplified amplitude of oscillations in the solution, unveiling the captivating complexities of the ecosystem's response to alterations in the growth rate of the top predator.



Figure 6. Impact of β_2 on the population dynamics for two different growth rate values

Figure 7-a shows a training process for an artificial neural network (ANN) to model the impact of a specific cannibalism rate on population dynamics in two scenarios. The ANN's accuracy is measured by the mean squared difference between its predictions and actual population data. The best validation performance was achieved at epoch 1000, indicating the model's optimal configuration. The model's ability to generalize and develop during training suggests it may be resilient. As a result, the ANN may help understand and manage populations impacted by cannibalism by making accurate predictions on population dynamics under various cannibalism scenarios. Throughout one thousand training epochs, the train, validation, and test datasets' mean squared errors (MSEs) are displayed in Figure 7-b. The effect of a single parameter, c=0.013, on population dynamics is the target of the ANN's training. When an ANN achieves peak performance on a validation set, it can reliably predict outcomes in novel situations. Epoch 1000 is optimal since ANN learns and generalizes the most effectively



Figure 7. Training process of an artificial neural network (ANN) for the impact of two different cannibalism (a) c =0.03421 and (b) c=0.013421 on the population dynamics

Figure 8-a shows how an artificial neural network (ANN) is trained to account for the effect of μ =0.90085 on population dynamics for two distinct harvesting rates. Epoch 1000 was the sweet spot for the ANN's performance on the validation dataset; at this time, the MSE was 2.6785e-09. Triangular in shape, the graphic depicts three curves: training, validation, and test. While avoiding overfitting, reducing the MSE on the training dataset should be the goal. An upward trend in the validation curve at epoch 800 indicates that overfitting is being monitored. The relatively low MSE shows good generalization to new data around 10^{-5} , measured by the test curve, which evaluates the ultimate performance. A model of the effect of a specific harvesting rate on the dynamics of a fish population is shown in Figure 8-b, which shows the training process of an ANN. The graph shows the ANN's mean squared error (MSE) on training, validation, and test MSE on the validation dataset. The MSE on training data decreases rapidly over the first few epochs, while on validation data, it increases slightly after 200 epochs, indicating overfitting.



Figure 8. Training process of an artificial neural network (ANN) for the impact of two different harvesting rates (a) μ =0.90085 and (b) μ =0.10085 on the population dynamics

Figures 9-a and 9-b show a simulation of a predator-prey system with cannibalism, depicting the population of basal prey changes over time for different cannibalism rates. This figure details the transition state of cannibalism rate, Gradient, and Mut. The simulation runs smoothly with no errors. The two plots show the population of basal prey for different cannibalism rates, with higher rates leading to a lower population of basal prey. This is a typical result in predator-prey models. Figures 10-a and 10-b show a study on population dynamics in a system affected by harvesting using a hybrid artificial neural network (ANN) approach. The model predicts system behavior for different harvesting rates with high accuracy. The population density decreases with higher harvesting rates, as harvesting removes individuals. The model's performance is evaluated through validation checks over time, with zero validation checks for both rates. The image also provides additional information about the ANN model, including Gradient, mu value, and epoch. Overall, the ANN model accurately models the population dynamics of a system affected by harvesting



Figure 9. Results of the transition state of cannibalism rate of basal prey (a) c=0.03421 and (b) c=0.013421 on the population dynamics



Figure 10. Results of the transition state on the population dynamics for two different harvesting rates (a) μ =0.90085 and (b) μ =0.10085

The error histogram in Figures 11-a and 11-b is a visual representation of the error distribution between the target values and the output values of an ANN model for predicting the cannibalism rate of basal prey. The histogram has 20 bins, with the x-axis representing the range of errors and the y-axis showing the number of instances within each bin. The errors are mostly concentrated around zero, indicating the ANN model's good performance. However, there is a small tail of errors on both the left and right sides, indicating significant differences from the target values. The error histogram in Figures 12-a and 12-b is a graphical representation of the distribution of errors in a data set, illustrating the differences between the target values and the actual outputs of a training program. The histogram shows the number of errors within each of the 20 bins, with the zero-error line at the center. The training data has the most errors around zero, with fewer errors as the error magnitude increases. The validation and test data have similar error distributions to the training data, indicating that the training program is not overfitting. The histogram is useful for visualizing error distribution and tracking training progress.

Figures 13-a and 13-b display four graphs illustrating the fit of a model for predicting the target variable (Y-T) against the actual target variable (Target). The model's fit is better when points are closer to the diagonal line. The top left graph shows the model perfectly fitting the training data, while the top right graph shows it doesn't. The bottom left graph shows the model fits the test data, and the bottom right graph shows it fits the combined data. The regression analysis shows the model fits the data well for both cannibalism rates. The Figures 14-a and 14-b show four graphs related to the training and validation of a neural network used for regression analysis. The top left graph shows the relationship between predicted and actual target values during the training phase, with all data points lying on the diagonal line. The top right graph shows the relationship between predicted and actual target values during the validation phase, with all data points close to the diagonal line. The bottom left graph shows the relationship between predicted and actual target values on a separate test dataset, with all data points close to the diagonal line. The bottom right graph combines data from all three phases into a single plot, indicating a good overall fit of the neural network model to the data. However, an R-value of 1 in all cases might suggest overfitting, where the model fits the training data too closely but may not generalize well to new data.



Figure 11. Histogram of error analysis for cannibalism rate of basal prey (a) c=0.03421 and (b) c=0.013421on the population dynamics for two different cannibalism rate



Figure 12. Histogram of error analysis of (a) = μ =0.90085 and (b) μ =0.10085on the population dynamics for two different harvesting rates



Figure 13. Regression analysis using the target of cannibalism rate of basal prey (a) c=0.03421 and (b) c=0.013421



Figure 14. Regression analysis from using the target of (a) = μ=0.90085 and (b) μ=0.10085on the population dynamics for two different harvesting rates

In this research, we use an artificial neural network (ANN) to predict and understand how the food chain model will act in the future. With the use of ANNs, we can better understand the interconnectedness of the tri-trophic food chain and predict how it will react in various scenarios. Particular insights and predictions offered by the ANN are listed below:

Pattern Recognition and Prediction: Artificial neural networks are excellent at identifying trends in data, and as a result, they are good at predicting future occurrences based on historical data. For example, the ANN can detect the trends of population dynamics and predict future occurrences within the framework of our food chain model. If we change the important characteristics like the prey growth rate, the predator predation rate, and the cannibalism intensity, it would be able to predict the long-run influence on all populations on all trophic levels.

Scenario Analysis: The ANN makes it possible to carry out a scenario analysis by simulating numerous hypothetical situations and assessing their outcomes. For instance, possible impacts of degrees of harvesting pressure on populations of Apex predators can be analyzed, and the capacity of the food chain model to adapt to radical alterations in climatic conditions can be evaluated. As a result of this, we can anticipate the potential barriers and create adaptable ways to handle them.

5- Conclusion

To better understand the complex dynamics of primary prey population cannibalistic interactions and the apex predator's selective harvesting techniques, we have developed a new tri-trophic food chain model. Our analysis, grounded in the rigorous application of the Routh-Hurwitz criteria and Lyapunov function analysis, has provided deep insights into the stability properties of coexistence equilibrium points within the system. Through visually compelling graphical representations, we have elucidated the significant influence of key parameters, such as intrinsic growth rates, cannibalism intensity and predation rates, on the overall dynamics of the system. Our integration of artificial neural networks into the analytical framework has further enriched our ability to gain profound insights and enhance predictive capabilities. One of the central contributions of our research lies in highlighting the critical aspects of conditional linear stability and global asymptotic stability concerning coexistence equilibrium points. Our findings underscore how variations in parameter values can induce complex oscillatory patterns in the temporal evolution of solutions, thus emphasizing the system's susceptibility to parameter perturbations and the potential emergence of dynamic complexities in real-world scenarios. Through the systematic flow of our methodology, as illustrated in the provided flowchart, we have navigated through the conceptualization, modeling, analysis, and visualization stages with precision and clarity. The complex relationship between species interactions and human harvesting techniques in ecological systems has been better understood with each step. Moving forward, our research enriches not only theoretical knowledge in the field of ecological modeling but also offers practical implications for effective ecosystem management strategies. By shedding light on the complex dynamics of tri-trophic food chains, our findings pave the way for informed decision-making aimed at promoting ecological stability and resilience in the face of environmental challenges.

Real-world food chains and ecosystems can be better understood and managed with the help of our study's findings. Ecological research and the creation of sustainable management strategies for resilient ecosystems are both improved by our use of mathematical modeling in conjunction with empirical data and sophisticated analytical methods.

6- Declarations

6-1-Author Contributions

Conceptualization, M.S.A. and A.E.; methodology, M.S.A.; software, A.E.; validation, A.M., A.E., and M.S.A.; formal analysis, A.M.; investigation, M.S.A.; resources, A.M.; data curation, A.M.; writing—original draft preparation, M.S.A.; writing—review and editing, A.E.; visualization, A.M.; supervision, M.S.A.; project administration, A.M.; funding acquisition, A.M. All authors have read and agreed to the published version of the manuscript.

6-2-Data Availability Statement

The data presented in this study are available on request from the corresponding author.

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Not applicable.

6-6-Informed Consent Statement

Not applicable.

6-7- Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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