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Enhancing Students' Conjecturing Skills Through RBL-STEM with Antimagic Coloring and Geometric Transformation in Batik Design

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Abstract

Students' conjecturing refers to the process wherein students make educated guesses or hypotheses about a problem, situation, or concept based on their prior knowledge, observations, and reasoning skills. It is an essential aspect of problem-solving and critical thinking skills. This research aims to enhance students' conjecturing thinking skills by implementing RBL-STEM using the (a,d)-edge antimagic coloring technique and geometric transformation to develop Batik motifs. A mixed methods approach was used, combining quantitative and qualitative analyses. Using an independent sample t-test, the quantitative analysis examined differences in conjecturing skills between the experimental and control classes. Qualitative analysis, through in-depth interviews, provided a triangulation of the quantitative findings. The results indicated a significant improvement in the students' conjecturing skills after the implementation of RBL-STEM. Statistical analysis at a confidence level of 5% showed a t statistic with Sig. (2-tailed) = 0.036 < 0.05, confirming the rejection of H0. The novelty of the study lies in integrating the (a,d)-edge antimagic coloring technique and geometric transformation into classroom research with STEM activities. These findings suggest that RBL-STEM has the potential to improve students' conjecturing abilities, particularly in the development of batik motifs rooted in local wisdom.

Keywords:

Conjecturing Thinking Skills;
RBL-STEM;
Local Wisdom-Based Batik Motifs;
(a, d)-Edge Antimagic Coloring;
Geometric Transformation.

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1- Introduction

Conjecturing thinking skills refer to the cognitive ability to formulate hypotheses, make educated guesses, and propose potential solutions or explanations to problems or situations based on available evidence or prior knowledge [1]. It involves the capacity to engage in creative and critical thinking processes to explore possibilities and generate new ideas without necessarily having all the information or evidence at hand. The ability to engage in conjecturing is frequently linked to inductive reasoning, which entails the formation of generalizations, or the making of predictions based on specific observations or patterns [2, 3]. Individuals who possess strong conjecturing thinking skills are adept at identifying connections, recognizing patterns, and extrapolating potential outcomes. These skills are particularly valuable in science, mathematics, engineering, and problem-solving contexts where complex and novel situations arise.

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They enable individuals to approach challenges with flexibility, creativity, and a willingness to explore multiple perspectives or solutions [4].

The possession of conjecturing thinking skills among students is still low. The weakness can be attributed to several factors: (1) Many educational systems prioritize rote memorization and regurgitation of facts over problem-solving and critical thinking skills. It is possible that students may be trained to memorize information rather than encouraged to think creatively and conjecture about possible solutions. (2) Lack of emphasis on inquiry-based learning: Traditional teaching methods often focus on delivering content through lectures and textbooks, leaving little room for students to engage in inquiry-based learning where they can explore, question, and conjecture about concepts and ideas, (3) Limited exposure to open-ended problems: Students are often presented with closed-ended problems that have clear-cut solutions rather than open-ended problems that require conjectural thinking and exploration of multiple possibilities. This lack of exposure to open-ended problems hinders the development of conjecturing skills, (4) Students may be hesitant to engage in conjectural thinking due to a fear of failure or a reluctance to take risks. The emphasis on getting the right answer rather than valuing the process of exploration and learning from mistakes can discourage students from engaging in conjecture, (5) Overemphasis on standardized testing: Educational systems that heavily rely on standardized testing tend to prioritize memorization and recall of information over higher-order thinking skills like conjecturing. As a result, students may focus on exam performance rather than developing their conjecturing abilities, (6) Limited opportunities for creative exploration: Schools may have rigid curricula and schedules that leave little time for students to engage in creative exploration and conjecture. The pressure to cover a vast amount of content within a limited timeframe can stifle opportunities for students to think critically and conjecture about concepts.

Various attempts to implement learning models aimed at improving students' conjecturing thinking skills face some significant challenges. Since, conjectural thinking involves the ability to make informed guesses, hypothesize, and engage in speculative reasoning. It's a higher-order cognitive skill that can be difficult to teach directly and assess accurately [5]. Secondly, evaluating conjecturing thinking skills often involves subjective judgment, as there may not be clear-cut right or wrong answers. This subjectivity can make it challenging to measure and assess students' progress effectively. Thirdly, designing reliable and valid assessment methods for conjecturing thinking skills can be challenging. Multiple-choice tests and standardized assessments may not capture the nuanced and qualitative aspects of conjecturing thinking effectively [6]. This is also the case, some students may be from diverse cultural and linguistic backgrounds, and thus we need to design inclusive learning models for improving their conjecturing thinking skills. Fourthly, teaching to improve the students’ conjecturing thinking skills often requires time for exploration, experimentation, and reflection. However, time constraints imposed by packed curricula and standardized testing schedules may limit opportunities for deep engagement with conjecturing thinking. Finally, some learning models aimed at improving conjecturing thinking skills may rely on technology tools and platforms. Access to reliable internet connectivity and appropriate technology resources can be a barrier, particularly in under-resourced schools and communities. Addressing these challenges requires a multifaceted approach.

Thus, in this study, we will apply RBL-STEM to improve the students’ conjecturing thinking skills and it will work effectively if the students are challenged with a specific task, namely developing ethnomathematics-based batik motifs using (a,d)-edge antimagic coloring and geometric transformation. RBL is a learning model based on constructivism, which emphasizes the construction of knowledge by individuals through direct experience and reflection. According to Dafik et al. (2023) [7], RBL uses several approaches, such as contextual learning (learning related to real contexts), authentic learning (learning that is relevant to real life), problem-solving (problem-solving), inquiry and discovery-based models, and hand on & mind on learning (learning that involves hands and minds). RBL emphasizes practice, learning from real situations, producing something through thinking processes, bringing research results into the learning process, and using research processes to solve complex problems [8]. In addition, RBL can improve analytical, evaluation, and synthesis to improve students' and lecturers' abilities in the assimilation and application of knowledge [9]. RBL is also considered to increase understanding of the role of research and innovation, which encourages students to think creatively and produce some results in the form of makerspace [10].

Meanwhile, STEM Education is a learning approach that involves four disciplines, namely Science, Technology, Engineering, and Mathematics [11, 12] The approach has a connection between problems that occur in real-world problems [13, 14]. The study of STEM subjects presents a distinctive set of characteristics that collectively facilitate more comprehensive problem-solving abilities in students. The integration of STEM in RBL can be carried out intensively to improve the students’ conjecturing thinking skills in developing into-mathematics-based batik motifs using (a,d)-edge antimagic coloring and geometric transformation. How does it work? Appelbaum (2023) describes the involvement of four characteristics of STEM, namely: (1) Science is the study of the laws and concepts that apply in nature. (2) Technology is the use of the Internet of Things (IoT) to solve problems and complete work. (3) Engineering is the knowledge to operate or design a procedure to solve a problem. (4) Mathematics is a science that links quantities, numbers, patterns, shapes, and space, requiring logical arguments without or accompanied by empirical evidence [15]. Previous studies have shown that the learning process under the implementation of RBL-STEM can improve the students’ thinking skills, Ridlo et al. [16] implemented the RBL-STEM learning model to enhance combinatorial thinking skills. The results obtained demonstrated that 45% of students were able to master combinatorial skills, and the average score achieved in student protest scores was 80.025. This suggests that the RBL-STEM learning model is effective in improving student combinatorial skills. Dafik et al. [7] conducted research on the application of the RBL-STEM learning model to improve students' meta-literation in solving wallpaper decoration problems using local antimagic coloring techniques. The study indicated the successful application of the RBL-STEM learning model, with 48% of students demonstrating very good metaliteration skills. Sumardi et al. [17] applied the RBL-STEM learning model integrated with JBatik software to enhance historical literacy by designing new batik models. This study demonstrated exceptional success, as evidenced by the increase in the average score of students reaching 83.50 with very high achievement category criteria. Sumarno et al. [18] conducted research using the RBL-STEM model to improve student financial literacy, which has a significant impact on student consumptive behavior. The outcomes of this research demonstrated remarkable success, as evidenced by the composite reliability (CR) value exceeding 0.7 and an average variance extracted (AVE) value greater than 0.5. These findings signify a substantial impact of implementing the RBL-STEM model on enhancing student financial literacy. Furthermore, Ridlo et al. [10] obtained satisfactory results in their research, which involved the development of teaching materials using machine learning combined with the RBL-STEM model to improve student computational skills. This is indicated by the existence of a very significant effect, as demonstrated by an alpha value of 0.003, which indicates a positive influence on teaching materials tested on students integrated with the RBL-STEM Learning model. A review of the extant literature pertaining to the RBL-STEM Model reveals that it has achieved a relatively high success rate. It is therefore hypothesized that the Research Based Learning Model with a STEM approach in the learning process will foster students' comprehensive ability to obtain new breakthroughs as a solution to a problem.

Furthermore, by graph coloring, we mean the process of assigning color to the elements of a graph, either vertices or edges, such that no two adjacent elements receive the same color [19]. The k-coloring of a graph is a proper coloring that involves k colors. A graph with a k-coloring is said to have a chromatic number of k [20, 21]. Meanwhile, antimagic labeling is the process of assigning labels on the elements of a graph, either vertices or edges, starting from the number 1 up to the number of vertices or edges, in such a way the sum of the labels of two adjacent vertices or edges has different weights [22]. Meanwhile, it is called to be a local antimagic labelling if only two adjacent vertices or edges have different weights. If the weights of local antimagic labelling are used to color the graph elements, then this concept eventually becomes a local antimagic coloring [23]. Moreover, if the set of colors taken over from the edge weight form an arithmetic sequence with initial value a and different d and a,d $\geq$1, then the local antimagic coloring of the graph is said to be local (a,d)-vertex or edge antimagic coloring [24]. Meanwhile, geometric transformations refer to the processes of altering the position, size, orientation, or shape of objects in a geometric space [25]. These transformations are fundamental concepts in mathematics, computer graphics, computer vision, and various engineering disciplines. Some common geometric transformations include translation, rotation, dilatation, reflection, scaling, shearing, and affine transformation. Geometric transformations find applications in various fields such as computer graphics for rendering images, computer vision for recognizing objects, robotics for motion planning, and mathematics for solving problems related to geometric graph theory. They are also widely used in game development, simulation, animation, and CAD (Computer-Aided Design) systems. In this study, students will be given some tasks to develop ethnomathematics-based batik motifs using local (a,d)-edge antimagic coloring technique together with geometric transformation concept. The chromatic number $\chi_{(le(a,d))}$ (G) is a minimum number of colors taken over all coloring induced by local antimagic edge labelling of G graph [26].

By ethno-mathematics-based batik motifs, we mean the use of mathematical principles, patterns, and concepts inspired by the cultural heritage and traditional designs found in certain ethnic groups or societies, particularly in the context of creating a motif on batik textiles or hand-drawn batik. Ethnomathematicss is a field that explores the relationship between mathematics and culture, particularly within indigenous or ethnic groups [27]. It involves studying how various societies develop mathematical ideas, concepts, and practices within their cultural contexts. As we know, batik is a traditional method of dyeing cloth, particularly practiced in Indonesia. On the second of 2009, it was an important day for Indonesian culture, especially batik. Since that day batik has been recognized as a world cultural heritage originating from Indonesia. The United Nations Educational, Scientific and Cultural Organization (UNESCO) and the United Nations (UN) have designated batik as a Masterpiece of the Oral and Intangible Heritage of Humanity in Indonesia. According to the UNESCO website, techniques, symbolism, and culture related to batik are considered inherent to the Indonesian culture. In fact, UNESCO considers Indonesian people to interpret batik from birth to death.

In the context of batik, motifs can represent cultural symbols, nature, folklore, or abstract geometric patterns [17]. We can find a big type of Indonesian batik since every district has specific batik motifs. For example, we have Keris batik, Sogan, Tujuh Rupa, Gentongan, Keraton, Parang, Kawung, Sidomukti, Truntum, Mega Mendung, Sekar Jagad, Sido Mukti, Gringsing, sekar jagat, as well as Jember batik. When we develop a type of Indonesian batik influenced by mathematical principles and concepts, we say "ethnomathematics-based batik motifs". This fusion of mathematics and cultural heritage results in unique and meaningful designs that reflect both mathematical aesthetics and cultural symbolism. Figure 1 shows some examples of ethnomathematics based batik motifs. The figure tells us how to develop ethnomathematicsal based Jember batik motifs using local (a,d)-edge antimagic coloring techniques together with the geometric transformation concept. To design ethnomathematics based Jember batik motifs, first we obtain the local (a,d)-edge antimagic coloring graphs, secondly we use geometric transformation concepts such as reflection, translation, rotation, and dilatation, and last we add tobacco leaves as the combination of the cultural heritage of Jember district, see Figures 1 and 2.

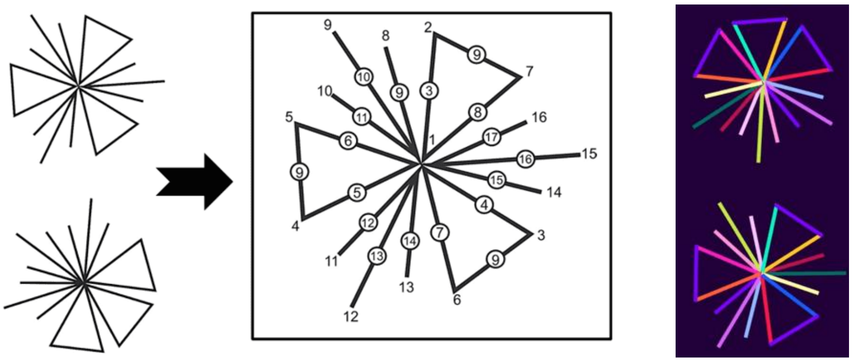


Figure 1. The process of local (a,d)-edge antimagic coloring transformed into Batik Motif

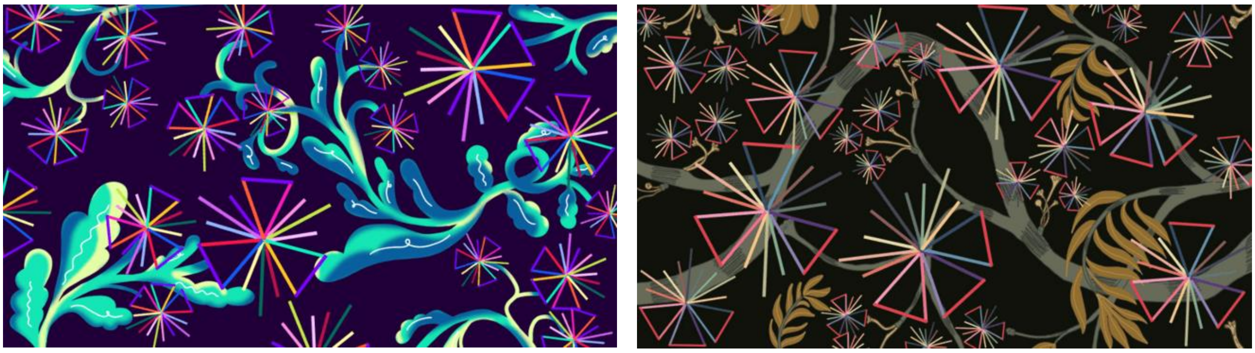


Figure 2. The Ethnomathematicss based Jember batik motifs using local (a,d)-edge antimagic coloring combined with reflection, translation, rotation, and dilatation

The combination of the concept of (a,d)-edge Antimagic Coloring and Geometric Transformation with RBL-STEM is seamlessly integrated by analyzing the problems that arise in determining the batik pattern with cultural character that will be developed. Furthermore, students are invited to research the background of batik motifs and the characteristics of batik through the integration of research activities in the classroom. The STEM aspect contributes to the process of directed student activity by carrying out activities related to science, technology, engineering and mathematics. The complete description of the integration of the concept of (a,d)-edge Antimagic Colouring and Geometric Transformation with RBL-STEM is shown in Figure 3.

Batik motifs are typically distinguished by their cultural and local provenance, yet they have evolved in tandem with technological advancements, giving rise to numerous contemporary batik designs. The genesis of local wisdom batik patterns necessitates a substantial degree of creativity, entailing the amalgamation of diverse batik patterns to yield visually appealing colour combinations that define the distinctive character of the resulting batik. The employment of the (a,d)-edge Antimagic Colouring and Geometric Transformation technique has been demonstrated to facilitate the development of novel batik patterns by students within the context of RBL-STEM.

Science

Developing local wisdom batik motifs using a combination of bright colours with beautiful gradations used for the motifs and considering the use of natural-based colours.

Technology

The utilisation of software for the design of batik motifs is a recent development, with the aim of creating batik patterns that embody local wisdom, software applications.

Engineering

The use of the (a,d)-edge Antimagic Colouring and Geometric Transformation in developing Batik motifs upon local wisdom.

Mathematics

Studying the number of permutations that can be developed from (a,d)-edge Antimagic Colouring and the geometric transformation obtained to design batik motifs.

Figure 3. The STEM Problems in designing Batik Motifs using (a,d)-edge Antimagic Colouring and Geometric Transformation

By considering the STEM aspects that have been integrated with (a,d)-edge Antimagic Colouring and Geometric Transformation, the RBL-STEM model can be seamlessly implemented in learning activities using the six learning steps described in Figure 4.

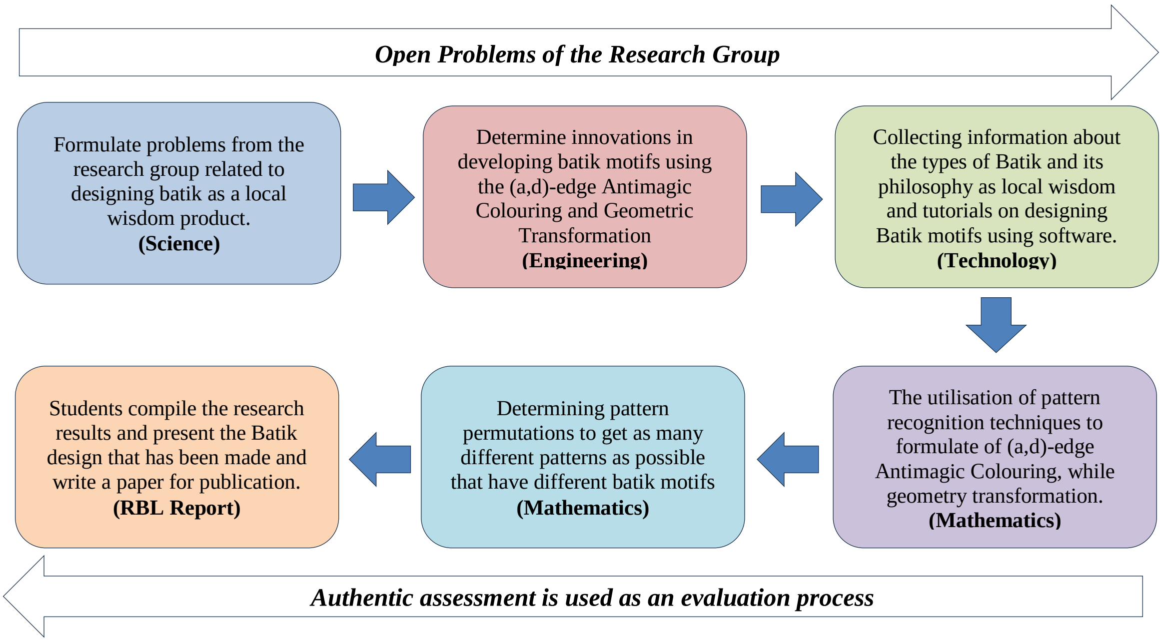


Figure 4. The syntax of the RBL-STEM approach in designing Batik motifs using (a,d)-edge Antimagic Coloring and Geometric Transformation

The research gaps identified in studies on conjectural thinking skills pose significant challenges in achieving maximum levels among students. Despite the various influences on learning, the results of previous studies indicate that there are still potential opportunities for improvement in effectively enhancing students' conjectural thinking skills. Ibrahim et al. [28] examined students' conjecturing ability, finding that 84% of students had low conjecturing ability. Only 2% of students demonstrated good conjecturing ability, while 12% exhibited average conjecturing ability. This suggests that students' conjecturing ability remains in the low category. Similarly, Sutarto et al. [29] examined students' conjecturing ability using problem-based learning models, resulting in the ability to generalize conjecture, which remained low with scores of 53.85 and 65.3. The ability to validate conjecture is also at a low level, with scores of 34.62, 50.00, and 42.31, respectively. These findings indicate that students' conjecturing ability is still in the low category. Furthermore, Wardani et al. [30] examined the students' conjecturing ability on one of the graph theory subjects, namely antimagic vertex dynamic coloring, using a research-based learning model. The results indicated that the students' conjecturing ability was at a low level, with only 27% of students achieving a high level of ability. This was reinforced by the students' post-test results, which only reached an average score of 69.58. The findings of this study suggest the necessity for a more effective, efficient and robust learning model that can facilitate students in enhancing their conjunctive thinking ability above the moderate level.

Based on the above background, the research questions in this study are as follows: (1) How is the process and result of development learning material based on RBL-STEM makerspace to promote conjecturing thinking skills in developing ethnomathematicss based batik motifs using (a,d)-edge antimagic coloring and geometric transformation? (2) Can the implementation of the RBL-STEM promote the students' conjecturing thinking skills in developing ethnomathematicss based batik motifs using (a,d)-edge antimagic coloring and geometric transformation?

2- Material and Methods

2-1- Types of Research

This study adopted a mixed methods framework, which included both quantitative and qualitative research methods to provide a comprehensive understanding. Quantitative research was conducted with the aim of assessing the significance of differences in students' reasoning skills through statistical analysis using independent sample t-test, to obtain numerical data that can be compared objectively between groups in experimental and control classes. Meanwhile, the qualitative approach involved the development of RBL-STEM-based learning materials [31], designed to support the improvement of students' thinking skills. This approach also included an in-depth analysis of the interviews to explore students' insights strengthening the quantitative data with the context and background of students' behaviors, as well as a detailed description of the phases of students' conjectural thinking skills [32], covering the initial process to the final achievements in the study. With the combination of these two approaches, the research aims to provide holistic and in-depth data, both in terms of numbers and qualitative understanding, in order to develop recommendations that are more applicable and relevant for the development of research-based and STEM education.

The triangulation process in mixed methods framework plays an important role in improving the validity and reliability of findings by comparing and integrating data from different sources or methods [33]. In the context of triangulation to confirm or challenge the findings of quantitative analyses, the first step is to compare quantitative and qualitative results. If the qualitative data, in the form of interview data, supports the patterns found in the quantitative analysis, the findings can be considered more valid. Conversely, if the qualitative data provides conflicting insights, more in-depth analyses are needed to understand the contradictions. Triangulation also helps identify anomalies in the quantitative data, such as small groups that do not conform to trends, for which qualitative data can provide additional explanations [34]. In addition, qualitative data often provides additional context or details that broaden the understanding of the quantitative results expressed in the interview results. When quantitative and qualitative results align, it strengthens the reliability of the findings; however, if there are discrepancies, triangulation can evaluate the accuracy of the data collection instrument.

The triangulation process involves several main steps, namely collecting data using quantitative and qualitative methods, analyzing both data separately, comparing the results, and integrating the findings to answer the research questions holistically [35]. As this study used a test to measure conjecturing thinking skills (quantitative data) and interviews to explore their perceptions of learning by using RBL-STEM to improve students’ conjecturing skills (qualitative data), triangulation can confirm the improvement of students' conjectural thinking skills as well as their positive perceptions. However, triangulation can also challenge the findings if there are students whose scores increase and feel that learning is still difficult. Thus, triangulation allows researchers to understand the data in depth, provide strong justification for the findings, and ensure accurate and reliable interpretations.

2-2- Data Collection and Procedure

The data collection procedure was conducted in accordance with the tenets of research ethics and was duly approved by the FKIP University of Jember Research Ethics Committee Number: 6524/UN25.5.1/LL/2023, The data collection technique was carried out in the following steps: 1) The researcher prepared a validated semester lesson plan which was then used in the Geometric Graph Theory course, 2) The researcher conducted a pre-test to assess the initial students' conjecturing thinking skills, 3) The researcher conducted a learning cycle on designing the Batik motifs by using local edge antimagic coloring with the RBL-STEM learning materials, and observed the students' learning activities with an observation sheet, 4) In each session, students applied local edge antimagic coloring to the discovered graphs. They then used the RBL-STEM learning materials to design and solve the Batik motifs. 5. At the end of the learning cycle, students were given a post-test to assess their conjecturing skills. In order to assess the students' conjecturing thinking skills, the researcher conducted a post-test. Furthermore, a conjecturing thinking skills questionnaire was distributed to all respondents, and interviews were conducted with selected respondents in order to visualize the conjecturing thinking skill in portrait phase format. The global research design can be illustrated in Figure 5.

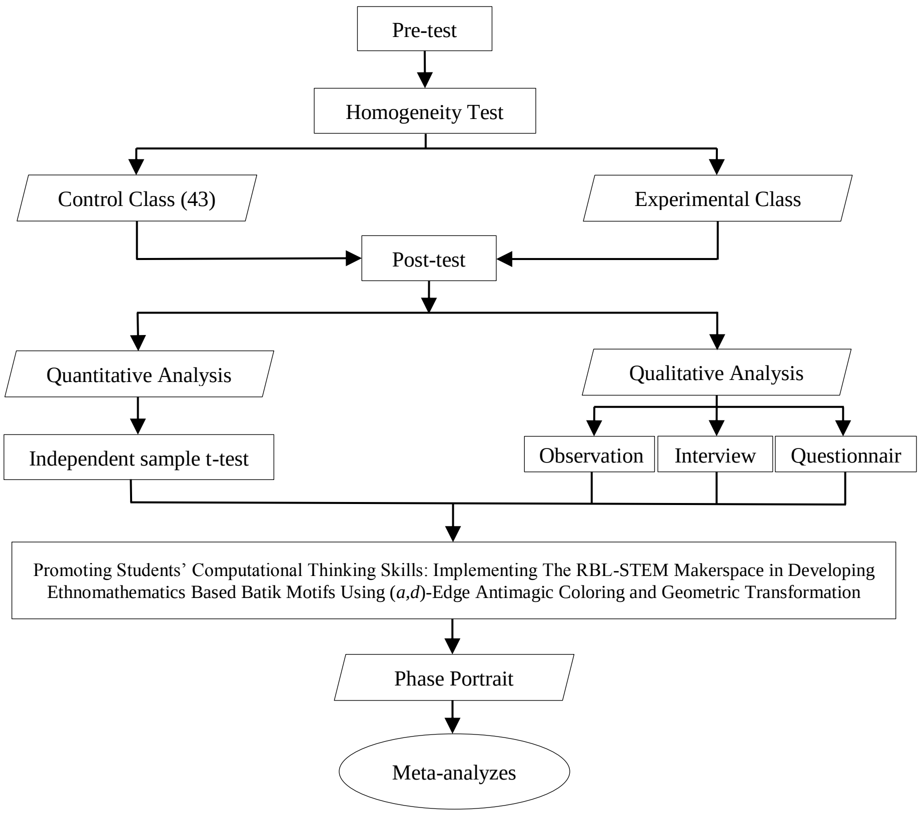


Figure 5. The flow chart of quasi-experimental design

2-3- Participants

This study took place at University of Jember, Indonesia. The selection of experimental and control classes was carried out using purposive sampling technique, which is a non-probability sampling method used to select samples based on certain criteria relevant to the research objectives [36]. The first step in this technique is to clearly determine the research objectives, namely the use of the RBL-STEM model to improve students' conjecturing thinking skills. The sampling method was employed, 43 students were identified for the control class and 44 students for the experiment class within the Mathematics Education program. Geometric Graph Theory course, segmented into two distinct classes each semester, had students averaging 22.5 years of age. These individuals were stratified into lower and higher ability groups according to their pre-test performance, measured in logits, with 61.5 serving as the threshold score for this division. The course was taught in Indonesian and included two lecture sessions. Students exhibited homogeneity in cultural background and an equivalent level of preliminary knowledge concerning the course. Given that the curriculum required students to have completed the Number Theory course prior to Geometric Graph Theory, both selected classes were deemed appropriate for the control and experimental groups, ensuring a balanced representation of the different ability groups. In order to control for confounding variables such as prior knowledge or learning ability, a number of strategies can be applied. Firstly, randomization can be employed to randomly divide participants into experimental and control groups, ensuring that confounding variables are distributed evenly. Secondly, sample homogenization can be achieved through the selection of participants who possess analogous characteristics with regard to confounding variables, such as the level of prior knowledge. Thirdly, matching can be applied by balancing participants between groups based on similarities in prior knowledge or learning ability. A pre-test can also be conducted to measure the initial condition of participants, the data of which is used for analysis or regrouping. The implementation of these strategies is intended to enhance the internal validity of the research and ensure the accuracy of the results.

2-4- Instruments

The instruments and educational resources employed in this research include a test for assessing conjecture thinking abilities, a semester lesson plan (RPS), student worksheets (LKM), observation sheets for activities, interview forms, and a survey [6]. The Likert scale, ranging from 1 to 5, was utilized to measure responses. All the tools underwent a comprehensive validation process, assessing content, format, and language, confirmed by experts. The validation panel consisted of a professor in mathematics education, two seasoned lecturers in the same field, a pair of mathematics educators, and an ethics committee from the University of Jember, Indonesia, overseeing the research's ethical standards.

The items included in the pre-test and post-test are derived from the indicators and sub-indicators associated with the conjecturing thinking skills. There are a total of seven indicators and fourteen sub-indicators associated with the conjecturing thinking skills. Specifically, these indicators and sub-indicators are employed to obtain data pertaining to students' conjecturing thinking skills in the context of solving open problems pertaining to local antimagic colouring. A local edge antimagic labeling of a graph is a bijection f from the vertex set of G to the number of vertices in G such that for any two adjacent vertices v and v', the edge weight w (vv')= f(v)+f (v'), where vv'∈E(G). Moreover, if we assign a color by the edge weight w (vv'), We have a local antimagic coloring of the graph. The chromatic number $\chi_{l\alpha}$ (G) is the minimum number of colors taken over all colorings induced by local antimagic edge labeling of G graph. Students are required to obtain $\chi_{l\alpha}$ (G) for specific graphs.

By these, we can derive the indicators and sub-indicators of conjecturing thinking skills in regard to the problem of local antimagic coloring presented in Table 1, as well as the indicators and sub-indicators of ethnomathematics in Table 2.

Table 1. Indicators and sub-indicators of conjecturing thinking skills

Indicator	Sub-indicator
Observing cases (CTS1)	A1: Identifying cases of ethnomathematics based batik motifs
	A2: Selecting a graph representation of ethnomathematics-based batik motifs
	A3: Determining the notation of the vertex and edge of the graph representation
	A4: Determining the cardinality of the graph representation
Organizing cases (CTS2)	B1: Assign the labels of (a,d)-edge antimagic coloring
	B2: Determining the(a,d)-edge antimagic coloring of the graph
	B3: Expanding the obtained (a,d)-edge antimagic coloring of the graph
Discovering and predicting pattern (CTS3)	C1: Expanding the obtained (a,d)-edge antimagic coloring for the generalized graph
	C2: Determining the chromatic number of obtained (a,d)-edge antimagic coloring the generalized of graph
Formulating conjectures (CTS4)	D1: Determining the pattern of (a,d)-edge antimagic coloring for the expanded graph
	D2: Determining the bijective function pattern of (a,d)-edge antimagic coloring for the expanded graph
Validating conjectures (CTS5)	E1: Testing the obtained bijective function pattern of (a,d)-edge antimagic coloring for the expanded graph by using a software application
Generalizing conjectures (CTS6)	F1: Determining the formula of the chromatic number of the (a,d)-edge antimagic coloring of the graph
Justifying conjectures (CTS7)	G1: Validating the determined chromatic number of (a,d)-edge antimagic coloring for the expanded graph

Table 2. Indicators and sub-indicators of ethnomathematics

Indicator	Sub-indicator
Number Concepts and Counting (E1)	The use of unique number systems in a culture.
	Different counting techniques, including the use of objects or body parts for counting.
	Traditional games involving counting or mathematical strategy.
Geometric Graph Theory and Shapes (E2)	The use of geometric shapes and patterns in traditional art and architecture.
	Concepts of space and proportion are used in cultural practices, like village layouts or weaving patterns.
	The symbolism is associated with certain geometric shapes in a cultural context.
Measurement and Quantification (E3)	Traditional measurement systems, include unique units of measure for length, weight, or volume.
	Locally developed measurement techniques and tools.
	The application of measurements in daily practices, such as agriculture, construction, or navigation.
Problem Solving and Logic (E4)	Unique problem-solving approaches in a cultural context, including the use of analogies or folk tales.
	Games that require logical or strategic thinking.
	Reasoning systems are used in oral traditions or justice practices.
Patterns and Relationships (E5)	The recognition and use of patterns in music, dance, or artwork.
	The study of relationships and proportions in cultural contexts, such as in crop planning or traditional astronomy.
	The use of mathematics in rituals and ceremonies, including timekeeping or calendrical calculations.

By those indicators, we developed the conjecturing thinking skills instruments. We did the validity and reliability tests on the instruments before using them. The conjecturing thinking skills test instrument consisted of 14 questions (4A, 3B, 2C, 2D, 1E, 1F, and 1G). Using the Pearson Correlation method, the two-tailed significance values for the majority of the items on the questionnaire indicate a score below 0.05, with the exception of one item that exceeds this threshold. In order to address the discrepancy in the significance level, an equivalent question was substituted and the test was repeated until the significance level was below 0.05. Furthermore, the reliability coefficient was obtained by calculating the Cronbach Alpha coefficient. The results showed that 0.682 (> 0.60). Thus, the conjecturing thinking skills instrument is reliable. The reliability calculation results are presented in Table 3.

Table 3. The result of the reliability instrument test of conjecturing thinking skills

Reliability Score	Cronbach's Alpha	Number of Items
0.682	0.674	14

2-5- Data Analysis Procedure and Research Hypothesis

The data was analyzed using both quantitative and qualitative techniques. The quantitative analysis sought to ascertain the significance of the difference in the achievement of students' conjecture-thinking skills between the control and experimental groups. In addition, qualitative methods were employed to analyze the results of in-depth interviews about the phase portrait form. A quasi-experimental design with a pre-test and post-test was utilized to analyze the discrepancy in the accomplishment of students' conjecture thinking capabilities between the control and the experimental groups. The analysis design is presented in Table 4.

Table 4. Quasi-Experimental Research Design

Class	Pre-Test	Treatment	Post-Test
Experiment Class	O1	X	O2
Control Class	O3	-	O4

Notes: O1, O3: Pre-test for the experimental class and control class; O2, O4 : Post-test for experimental and control classes; X: The implementation of RBL-STEM learning materials using the local antimagic coloring technique.

The data analysis was conducted to test the hypotheses proposed in this study as follows:

H_0= There is no significant difference in students' conjecturing thinking skills developing Ethnomathematics-based Batik Motifs using local antimagic coloring techniques between experiment and control classes.

H_1= There is a significant difference in students' conjecturing thinking skills developing Ethnomathematics-based Batik Motifs using local antimagic coloring techniques between experiment and control classes.

The data analysis on the pre-test was initially used to determine the initial condition of students' conjecturing thinking skills before the implementation of RBL-STEM. Subsequently, data analysis on the post-test was used to test the differences in students' conjecturing thinking skills between experiment and control classes at the Sig. (2-tailed) level using the inferential statistics student t-test. The hypothesis test will have the rejection of H0 if the value of Sig. (2-tailed) < 0.05, and it will imply the acceptance of H1. Furthermore, observation sheets and questionnaires with data sources from respondent answers were calculated using the Likert scale from 1 to 5 (1=poor, 2=fair, 3=good, 4=very good, 5=excellent).

The analysis of the portrait phase as a triangulation process is also described as a graph and adjacency matrix. Furthermore, after understanding the adjacency matrix for distances one and three, the RRA value is calculated from the Total Depth (TD), Mean Depth (MD), and Relative Asymmetry (RA) analysis. Total Depth (TD) is the total length of the path of observed sub-indicators, Mean Depth. $(MD) = \frac{TD}{n-1}$, and $RA = \frac{2(MD-1)}{n-2}$, then $RRA = \frac{RA}{GL}$, where $GL = 2\frac{n\sqrt{n}-2n+1}{(n-2)(n-1)}$ [37].

3- Result and Discussion

3-1- Pre-Test Data Analysis

The study employed two classes as the experimental and control groups, respectively. The study comprised 44 students from the experimental class and 43 students from the control class. In both classes, both the experimental and control groups were given pre-test questions to determine the initial conjecturing thinking skills of students from both groups. The data analysis of the pre-test results in the experimental class revealed that 9.75% of students demonstrated a medium level of conjecturing thinking skills, while 90.25% exhibited a low level of this skill. Conversely, the data analysis of the pre-test results in the control class revealed that 12.5% of students exhibited a medium level of conjecturing thinking skills, while 87.5% demonstrated a low level of such skills. An illustration of conjecturing thinking skills by surmising in both classes after being given pre-test questions is shown in Figure 6.

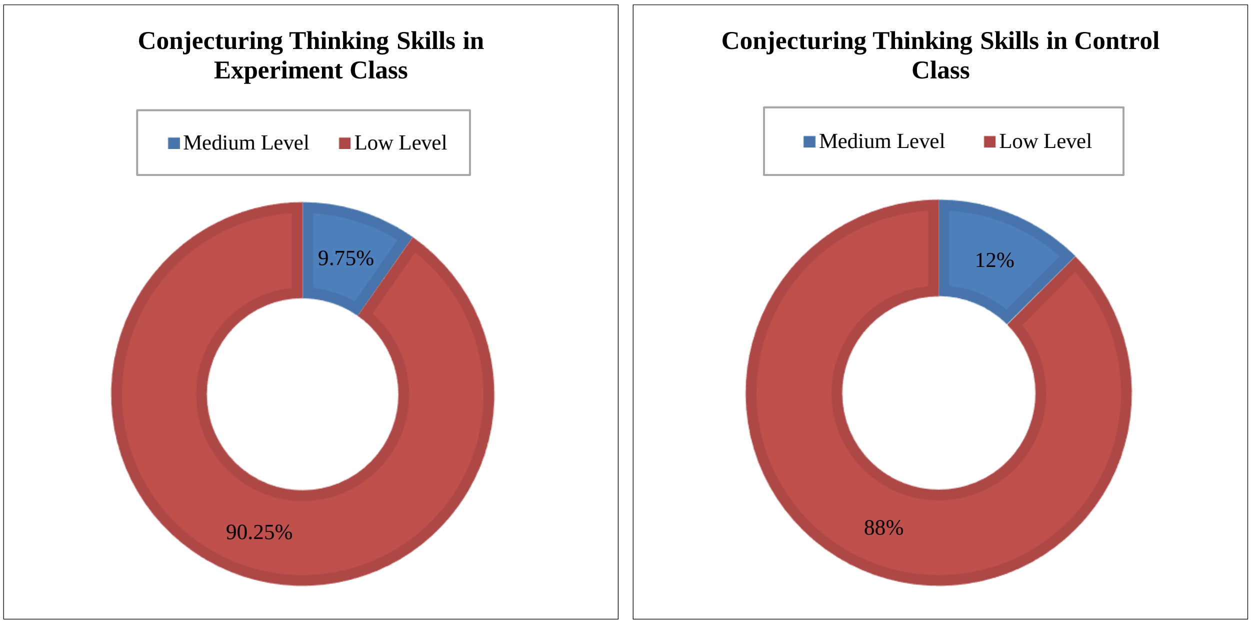


Figure 6. The illustration of conjecturing thinking skills by surmising in both classes after the pre-test

In addition, two separate samples of the t-test were conducted using the statistical software program SPSS. Prior to the execution of the two independent t-tests, it is essential to perform two preliminary tests, namely the normality test and the homogeneity test. The first statistical test is the normality test, which determines the distribution of the data from both classes, specifically whether the distributions are normally distributed.

Table 5 shows that both classes follow a normal distribution. This is demonstrated by the Asymp Sig (2-tailed) values: 0.142 for the experimental class and 0.130 for the control class, both of which are greater than the significance threshold of 0.05. Additionally, a homogeneity test was conducted to determine whether the variances of the two classes were consistent, confirming the assumption of homogeneity in the data.

Table 5. Results of Normality Test on Pre-Test

One-Sample Kolmogorov-Smirnov Test [a]			
		Experiment	Control
N		44	43
Normal Parameters [a, b]	Mean	50.24	50.38
	Std. Deviation	8.729	9.086
Most Extreme Differences	Absolute	0.121	0.123
	Positive	0.116	0.123
	Negative	-0.121	-0.105
Test Statistic		0.121	0.123
Asymp. Sig. (2-tailed)		0.142 [c]	0.130 [c]

a. Test distribution is Normal; b. Calculated from data; c. Lilliefors Significance Correction.

Table 6 indicates that both classes have equal variances, as shown by the Sig value of 0.764, which is greater than the significance threshold of 0.05. This confirms that the data from both classes are homogeneous. Following the normality and homogeneity tests, an independent sample T-test was conducted to determine whether there was a significant difference in the mean scores of the conjecturing thinking skills test between the two classes.

Table 6. Results of Homogeneity Test on Pre-Test

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
Pre Test	Based on Mean	0.091	1	79	0.764
	Based on Median	0.071	1	79	0.790
	Based on the Median and with adjusted df	0.071	1	78.89	0.790
	Based on trimmed mean	0.094	1	79	0.761

As shown in Table 7, the significance value (Sig) is 0.764, which is greater than the threshold of 0.05. This indicates that, prior to the start of instruction, there was no significant difference in the mean scores of the conjecturing thinking skills test between students in the experimental and control classes.

Table 7. Results of Two Independent Samples T-test on Pre-test

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Pre-test Score	Equal variances assumed	0.091	0.764	-0.066	79	0.947	-0.131	1.979	-4.071	3.809
	Equal variances are not assumed.			0.066	78.66	0.947	0.131	1.980	-4.073	3.811

3-2- Post-Test Data Analysis

The data analysis of the post-test results in the experimental class indicates that 22% of students can be classified as having a medium level of conjecturing thinking skills, while 78% of students can be classified as having a high level of conjecturing thinking skills. Conversely, the data analysis of the post-test results in the control class revealed that 30% of students exhibited a low level of conjecturing thinking skills, 57.5% demonstrated a medium level, and 12.5% exhibited a high level. An illustration of conjecturing thinking skills by surmising in both classes after being given post-test questions is shown in Figure 7.

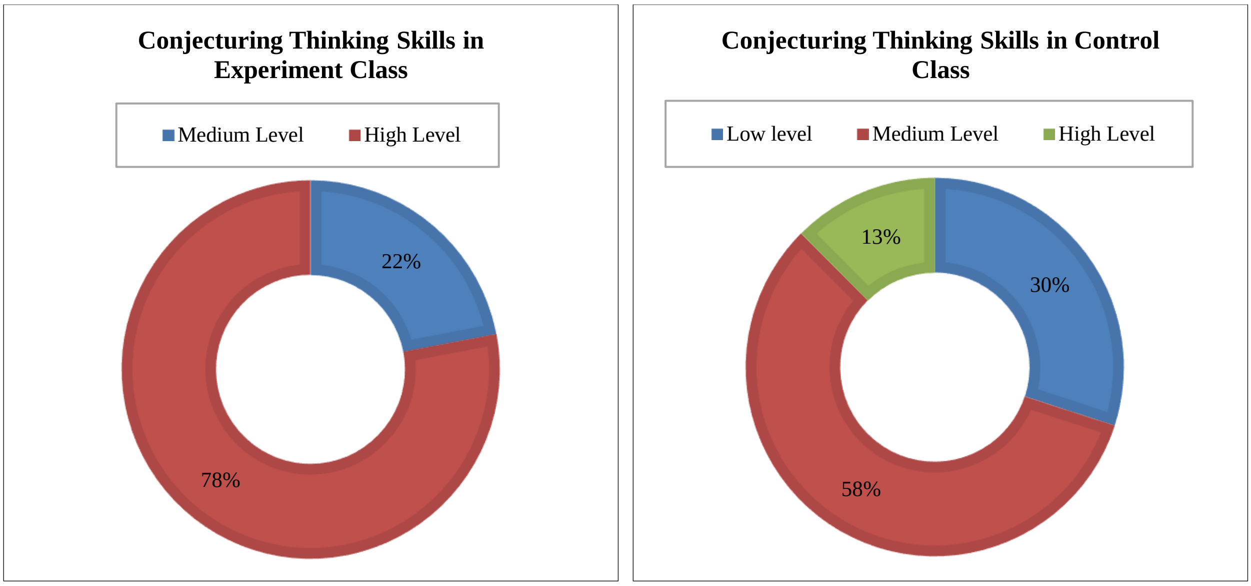


Figure 7. The illustration of conjecturing thinking skills by surmising in both classes after the post-test

Furthermore, two independent samples of the t-test were carried out using SPSS software. Prior to conducting the two independent sample t-tests, it is essential to perform two preliminary tests: the normality test and the homogeneity test. The initial statistical test is the normality test, which determines whether the data from both classes are normally distributed.

Table 8 indicates that both classes are normally distributed, as evidenced by the Asymp Sig (2-tailed) of the experiment class being 0.132 > 0.05 and the Asymp Sig (2-tailed) of the control class being 0.076 > 0.05. Moreover, the statistical test employed was the homogeneity test, which was used to ascertain whether the data variants of the two classes were homogeneous.

Table 8. Results of Normality Test on Post-Test

One-Sample Kolmogorov-Smirnov Test [a]			
		Experiment	**Control**
N		44	43
Normal Parameters [a, b]	Mean	85.24	66.50
	Std. Deviation	8.729	8.785
Most Extreme Differences	Absolute	0.122	0.132
	Positive	0.116	0.118
	Negative	-0.122	-0.132
Test Statistic		0.122	0.132
Asymp. Sig. (2-tailed)		0.132 [c]	0.176 [c]

a. Test distribution is Normal; b. Calculated from data; c. Lilliefors Significance Correction.

Based on Table 9, both classes have the same variance as the Sig value. Based on the mean obtained, it is 1.00 > 0.05. This indicates that both classes are homogeneous. After carrying out the normality and homogeneity tests, two independent sample t-tests were carried out to see if there was a difference in the mean score of the conjecturing thinking skills test between the two classes.

Table 9. Results of Homogeneity Test on Post-Test

Test of Homogeneity of Variances					
		Levene Statistic	**df1**	**df2**	**Sig.**
Post-test	Based on Mean	0.000	1	78	1.000
	Based on Median	0.011	1	78	0.915
	Based on the Median and with adjusted df	0.011	1	77.639	0.915
	Based on trimmed mean	0.001	1	78	0.990

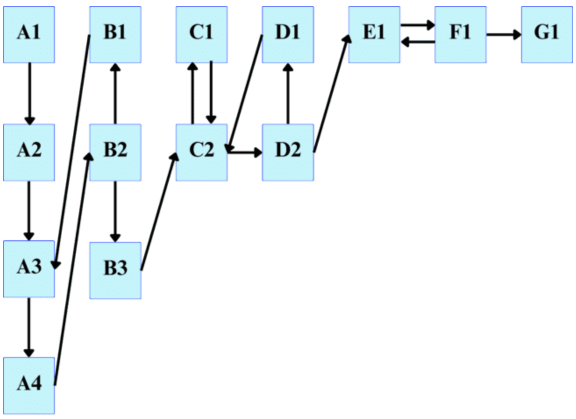
The significance Sig value is 0.021 < 0.05 based on Table 10. This indicates that after learning with RBL-STEM Makerspace-based Batik Motifs using (a,d)-edge Antimagic Coloring and Geometric Transformation, a statistically significant difference was observed in the average score on the conjecturing and thinking skills test between students in the experimental and control classes.

Table 10. Results of Two Independent Samples T-test on Post-test

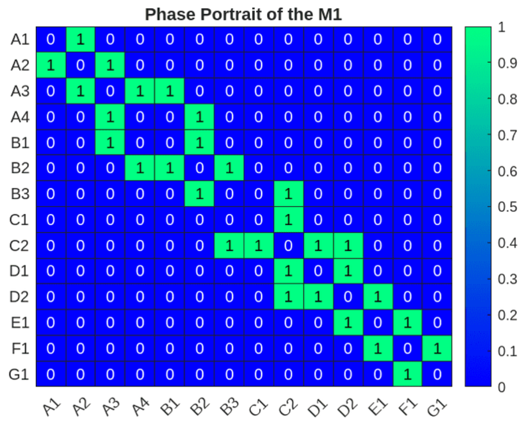
Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Pre-test Score	Equal variances assumed	5.562	0.021	14.287	78	0.000	25.125	1.759	21.624	28.626
	Equal variances are not assumed.			14.287	73.280	0.000	25.125	1.759	21.620	28.630

3-3- Phase Portrait

A phase portrait represents a student's conjecturing thinking process in an illustrated or diagrammatic form. The phase portrait of the students in this research is constructed from their conjecturing thinking skill patterns under the implementation of RBL-STEM Makerspace-based Batik Motifs using (a,d)-edge Antimagic Coloring and Geometric Transformation. Student-1 (M1) response indicates the first post-test result, which shows a high category of conjecturing thinking skills. Figure 8-a visually depicts the conjecturing thinking skill patterns of students who fall into high categories; while Figure 8-b shows the adjacency matrix of distance one from the graphical representation.



(a)



(b)

Figure 8. (a) Graph representation of M1, (b) Adjacency matrix of distance 1 of M1

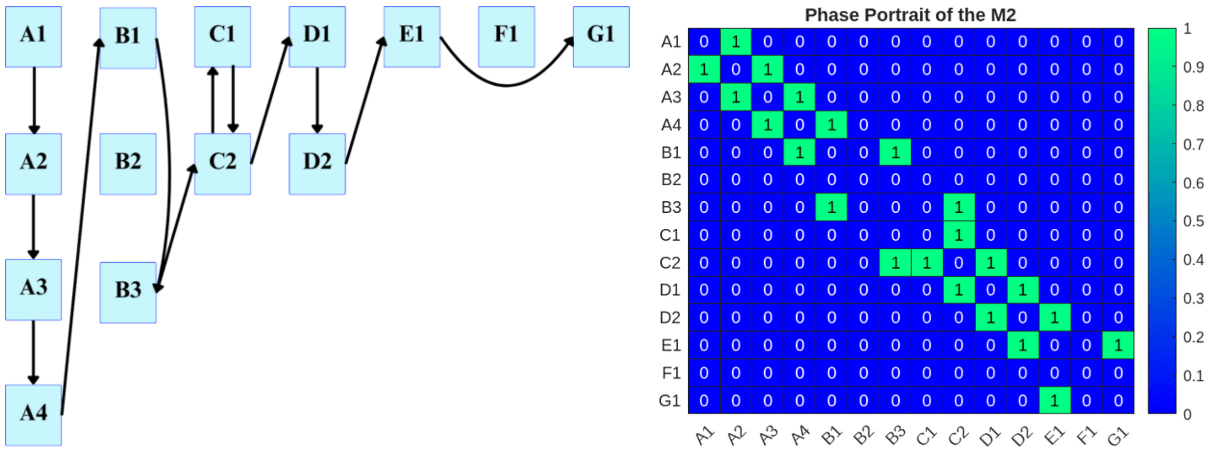
Next, we analyze the total depth (TD), mean depth (MD), relative asymmetry (RA), and real relative asymmetry (RRA) of the M1 conjecturing thinking process. The advantage of conducting this analysis lies in assessing students' conjecturing thinking skills through the lens of flow configuration. Total depth (TD) is the total number of path lengths of the observed sub-indicators, mean depth. $(MD) = \frac{TD}{n-1}$, real asymmetry $RA = \frac{2(MD-1)}{n-2}$, $GL = 2\frac{n\sqrt{n}-2n+1}{(n-2)(n-1)}$, and real relative asymmetry $RRA = \frac{RA}{GL}$. From this formula, the distribution of values in Table 11 is obtained.

Table 11. TD, MD, RA, and RRA values of M1 Phase Portrait

No.	Sub-indicators	TD	MD	RA	RRA	No.	Sub-indicators	TD	MD	RA	RRA
1	A1	135	10.38	1.56	4.80	8	C1	66	5.07	0.67	2.08
2	A2	114	8.76	1.29	3.97	9	C2	51	3.92	0.48	1.49
3	A3	95	7.30	1.05	3.23	10	D1	62	4.76	0.62	1.93
4	A4	46	3.53	0.42	1.30	11	D2	59	4.53	0.58	1.81
5	B1	46	3.53	0.42	1.30	12	E1	70	5.38	0.73	2.24
6	B2	47	3.61	0.43	1.33	13	F1	83	6.38	0.89	2.75
7	B3	48	3.69	0.44	1.37	14	G1	98	7.53	1.08	3.34

In Table 11, it is evident that sub-indicator A4 and B1 possess an RRA score of 1.30. The RA score associated with this particular sub-indicator is deemed optimal since lower RRA values, provided they are not negative, signify greater integrity and can be classified as favorable. Within this sub-indicator, student M1 experienced notable advantages in determining the cardinality of the graph representation and assigning the labels of (a,d)-edge antimagic coloring.

Next, we will discuss the phase portrait of student 2 (M2). M2 is a student with moderate conjecturing thinking skills. Figure 9-a shows the graphical representation of student M2 conjecturing thinking skills flow, while Figure 9-b shows the adjacency matrix of distance one from the graphical representation.



(a) (b)

Figure 9. (a) Graph representation of M2, (b) Adjacency matrix of distance 1 of M2

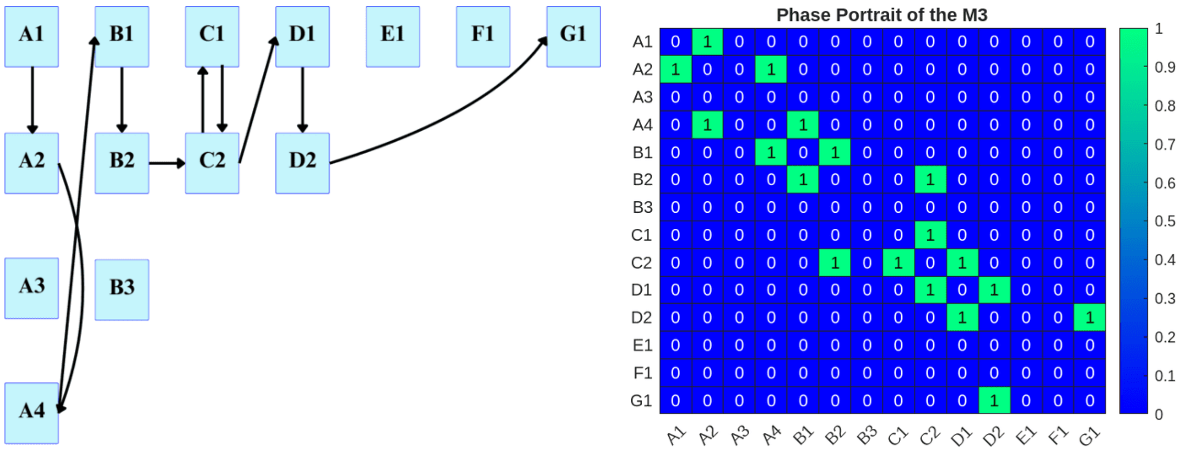
Next, we analyze the total depth (TD), mean depth (MD), relative asymmetry (RA), and real relative asymmetry (RRA) of M2 conjecturing thinking skills. The advantage of conducting this analysis lies in assessing students' conjecturing thinking skills through the lens of flow configuration. Total depth (TD) is the total number of path lengths of the observed sub-indicators, mean depth. $(MD) = \frac{TD}{n-1}$, real asymmetry $RA = \frac{2(MD-1)}{n-2}$, $GL = 2\frac{n\sqrt{n}-2n+1}{(n-2)(n-1)}$, and real relative asymmetry $RRA = \frac{RA}{GL}$. From this formula, the distribution of values in Table 12 is obtained.

Table 12. TD, MD, RA, and RRA values of M2 Phase Portrait

No.	Sub-indicators	TD	MD	RA	RRA	No.	Sub-indicators	TD	MD	RA	RRA
1	A1	62	5.63	0.92	2.74	8	C1	42	3.81	0.56	1.66
2	A2	52	4.72	0.74	2.20	9	C2	32	2.90	0.38	1.13
3	A3	44	4.00	0.60	1.77	10	D1	36	3.27	0.45	1.34
4	A4	38	3.45	0.49	1.45	11	D2	42	3.81	0.56	1.66
5	B1	34	3.09	0.41	1.23	12	E1	50	4.54	0.70	2.10
6	B3	32	2.90	0.38	1.13	13	G1	60	5.45	0.89	2.63
7	A1	62	5.63	0.92	2.74	14	C1	42	3.81	0.56	1.66

In Table 12, it is evident that sub-indicator B3 and C2 possess an RRA score of 1.13. The RA score associated with this particular sub-indicator is deemed optimal since lower RRA values, provided they are not negative, signify greater integrity and can be classified as favorable. Within this sub-indicator, student M2 experienced notable advantages in expanding the obtained (a,d)-edge antimagic coloring of the graph and determining the chromatic number of obtained (a,d)-edge antimagic coloring the generalized graph.

Next, we will discuss the phase portrait of student 3 (M3). M3 is a student with low conjecturing thinking skills. Figure 10-a shows the graphical representation of student M3 conjecturing thinking skills flow, while Figure 10-b shows the adjacency matrix of distance one from the graphical representation.



(a) (b)

Figure 10. (a) Graph representation of M1, (b) Adjacency matrix of distance 1 of M3

Next, we analyze the total depth (TD), mean depth (MD), relative asymmetry (RA), and real relative asymmetry (RRA) of M3 conjecturing thinking skills. The advantage of conducting this analysis lies in assessing students' conjecturing thinking skills through the lens of flow configuration. Total depth (TD) is the total number of path lengths of the observed sub-indicators, mean depth. $(MD) = \frac{TD}{n-1}$, real asymmetry $RA = \frac{2(MD-1)}{n-2}$, $GL = 2\frac{n\sqrt{n}-2n+1}{(n-2)(n-1)}$, and real relative asymmetry $RRA = \frac{RA}{GL}$. From this formula, the distribution of values in Table 13 is obtained.

Table 13. TD, MD, RA, and RRA values of M3 Phase Portrait

No.	Sub-indicators	TD	MD	RA	RRA	No.	Sub-indicators	TD	MD	RA	RRA
1	42	4.66	0.91	2.61	42	6	C1	30	3.33	0.58	1.66
2	34	3.77	0.69	1.98	34	7	C2	22	2.44	0.36	1.02
3	28	3.11	0.52	1.50	28	8	D1	26	2.88	0.47	1.34
4	24	2.66	0.41	1.18	24	9	D2	32	3.55	0.63	1.82
5	22	2.44	0.36	1.02	22	10	G1	40	4.44	0.86	2.45

In Table 13, it is evident that sub-indicator B2 and C2 possess an RRA score of 1.02. The RA score associated with this particular sub-indicator is deemed optimal since lower RRA values, provided they are not negative, signify greater integrity and can be classified as favorable. Within this sub-indicator, student M3 experienced notable advantages in determining the (a,d)-edge antimagic coloring of the graph and determining the chromatic number of obtained (a,d)-edge antimagic coloring of the generalized graph.

The insights gained from the in-depth interview process are closely related to the indicators of students' conjecturing ability in designing Batik motifs. This ability is trained through the application of the RBL-STEM learning model using the (a,d)-edge Antimagic Colouring and Geometric Transformation technique. During the interview process, students reported feeling facilitated in their learning activities due to the integration of research activities in the learning process. The STEM aspect also plays a role in helping students decipher important components that are related to designing Batik motifs with local wisdom. The close relationship between problem-solving in designing Batik, applying (a,d)-edge Antimagic Colouring and Geometric Transformation software techniques, and using supporting software to create aesthetically pleasing Batik designs is also highlighted. Furthermore, the students demonstrated a favorable response to the integration of cultural elements, underpinned by local wisdom, and mathematical techniques in the design process. This integration facilitates a comprehensive understanding of the role of mathematics in the design of batik motifs that reflect local wisdom.

3-4- NVIVO Analysis

The conclusion of the study involved using NVIVO software to qualitatively analyze variations in students' conjectural thinking skills. Researchers utilized a specific NVIVO tool called the Word Frequency Query to monitor the frequency of specific words in detailed interview transcripts with students. This analysis shows the compilation of commonly occurring terms in the interview data, as illustrated in Figure 11.

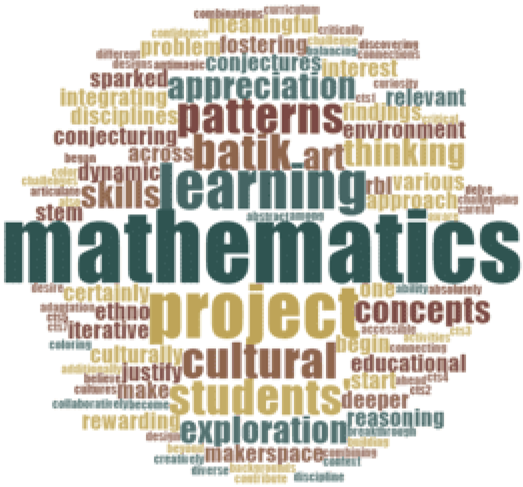
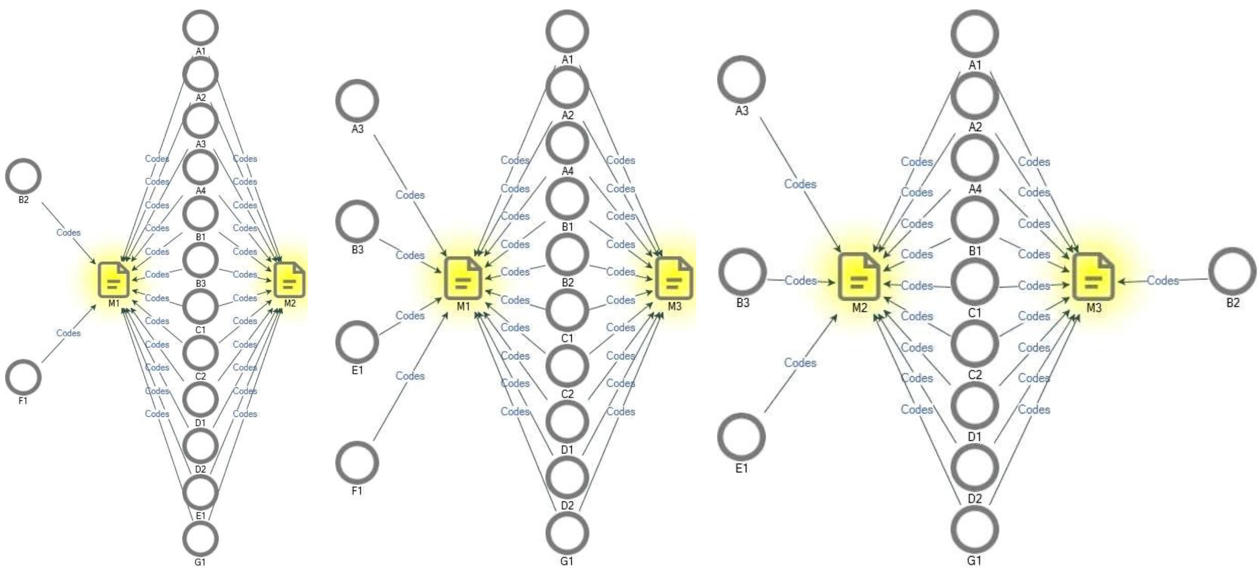


Figure 11. Column Word Cloud from Students

From now on, we will examine the comparative information provided by the three interviews. Comparative information is a useful feature of NVivo. In this section, the conjecturing thinking skills are divided into several sub-indicators. The first indicator is observing cases (CTS1) which have 4 sub-indicators, namely identifying cases of ethnomathematics-based batik motifs (A1), selecting a graph representation of ethnomathematics batik motifs (A2), determining the notation of the vertex and edge of the graph representation (A3), and determining the cardinality of the graph representation (A4). The second indicator is organizing cases (CTS2) which have 3 sub-indicators, namely assign the labels of (a,d)-edge antimagic coloring (B1), determining (a,d)-edge antimagic coloring of the graph (B2), and expanding the obtained (a,d)-edge antimagic coloring of the graph (B3). The third indicator is discovering and predicting pattern (CTS3) which has 2 sub-indicators, namely expanding the obtained (a,d)-edge antimagic coloring for the generalized graph (C1) and determining the chromatic number of obtained (a,d)-edge antimagic coloring the generalized of the graph (C2). The fourth indicator is formulating conjectures (CTS4) which have 2 sub-indicators, namely determining the pattern of (a,d)-edge antimagic coloring for the expanded graph (D1) and determining the bijective function pattern of (a,d)-edge antimagic coloring for the expanded graph (D2). The fifth indicator is validating conjectures (CTS5) which have 1 sub-indicator, namely testing the obtained bijective function pattern of (a,d)-edge antimagic coloring for the expanded graph by using a software application (E1). The sixth indicator is generalizing conjectures (CTS6) which has 1 sub-indicator, namely determining the formula of the chromatic number of the (a,d)-edge antimagic coloring of the graph (F1). Last, the justifying conjecture (CTS7) indicator also has 1 sub-indicator, namely validating the determined chromatic number of (a,d)-edge antimagic coloring for the expanded graph (G1).

In Figure 12-a, we observe the distinct sub-indicators that can be fulfilled by M1 and M2. Notably, sub-indicators B2 and F1 are achievable only by M1, highlighting differences between M1 and M2. Figure 12-b illustrates the specific sub-indicators that can be achieved by M1 compared to M3, where A3, B3, E1, and F1 are exclusive to M1. Similarly, Figure 12-c shows the differences in sub-indicators achievable by M2 and M3. Specifically, A3, B3, and E1 are attainable only by M2, while B2 is exclusive to M3.



(a) (b) (c)

Figure 12. The comparison between (a) M1 and M2, (b) M1 and M3, (c) M2 and M3

Moreover, we analyze the students' overall data and its relationship with the predefined categories. Additionally, we will present the classification outcomes from the interviews. Figure 13 displays the project map feature of NVivo. This project map aligns with our previous analysis, indicating that M1 meets all sub-indicators, whereas M2 and M3 do not fulfill certain sub-indicators.

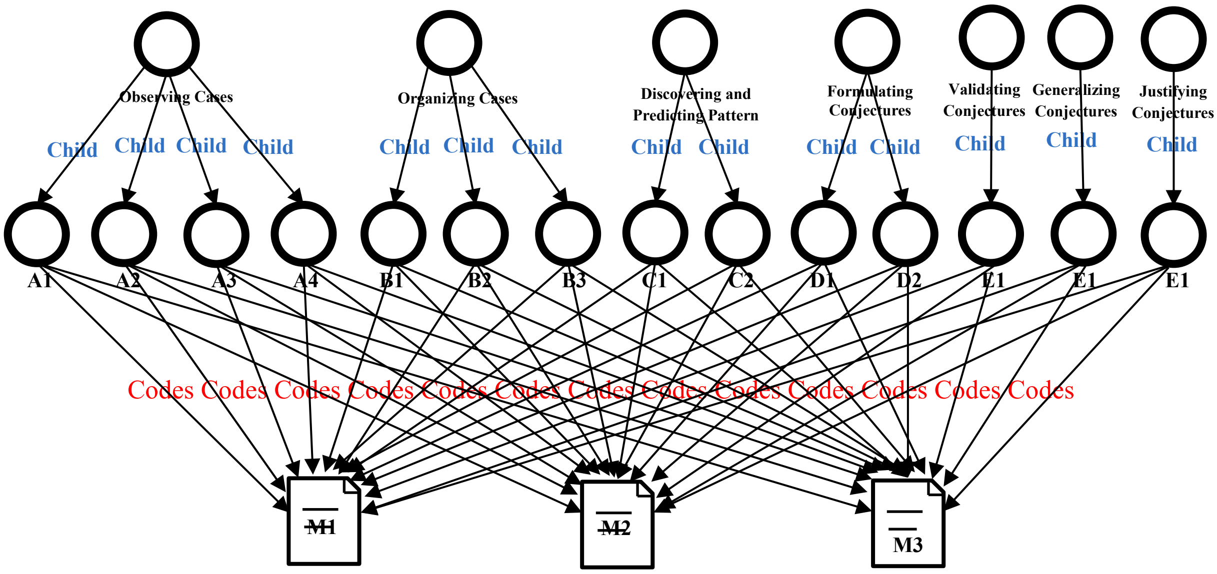


Figure 13. Project Map of Conjecturing Thinking Skills on M1, M2, and M3

3-5- Smart-PLS Analysis

The last, we deploy SmartPLS. The primary aim of using SmartPLS (Partial Least Squares Structural Equation Modeling) is to facilitate the analysis of complex relationships between observed and latent variables in various datasets, particularly when the data are non-normally distributed or when sample sizes are small. The software has been designed with the intention of providing a user-friendly graphical interface that simplifies the modeling process, making it accessible to both novice and experienced researchers. The software, SmartPLS, focuses on maximizing the explained variance of dependent latent constructs, making it especially useful in exploratory research where the goal is to predict and understand key drivers of outcomes. By offering features such as bootstrapping for reliability assessment, support for formative measurement models, and multi-group analysis, SmartPLS enables researchers to conduct robust analyses that inform strategic decision-making in areas such as marketing, management, and social sciences, thereby enhancing the quality and validity of empirical research findings.

As shown in Figure 14 shows the standard algorithm assessing several elements including the loading factor value, reliability, and average variance extracted (AVE). The loading factor value aims to confirm each sub-indicator's convergent validity, which is considered strong if the value is above 0.7. The specific loading factor values are 0.754 for the RBL-STEM indicator, 0.796 for the Ethnomathematics indicator, and 0.901 for the Conjecturing Thinking Skills (CTS) indicator. A comprehensive presentation of the loading factor values, reliability score from Cronbach's alpha, AVE, and composite reliability are provided in Table 14.

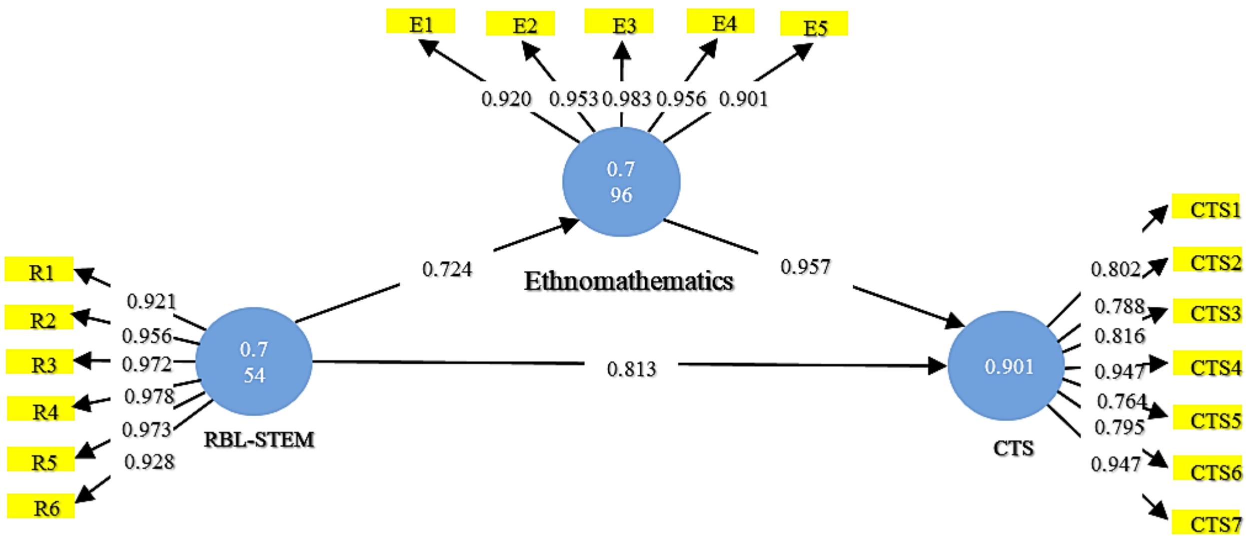


Figure 14. SEM-PLS Analysis

Table 14. Loading factor value, reliability Cronbach's alpha, composite reliability, average variance extracted

Indicator	Sub-indicator	Loading factor	Reliability Cronbach's Alpha	Composite Reliability	AVE
RBL-STEM	R1	0.921	0.754	0.791	0.847
	R2	0.956			
	R3	0.972			
	R4	0.978			
	R5	0.973			
	R6	0.928			
Ethnomathematics (E)	E1	0.920	0.796	0.867	0.827
	E2	0.953			
	E3	0.983			
	E4	0.956			
	E5	0.901			
Conjecturing Thinking Skills (CTS)	CTS1	0.802	0.901	0.749	0.843
	CTS2	0.788			
	CTS3	0.816			
	CTS4	0.947			
	CTS5	0.764			
	CTS6	0.795			
	CTS7	0.947			

Table 14 illustrates that all sub-indicators in the model exhibit loading factor values exceeding 0.7, indicating strong convergent validity. This indicates that each sub-indicator effectively measures its intended indicator. In addition to convergent validity, the model's assessment includes the examination of other metrics, such as Cronbach's alpha reliability, composite reliability, and average variance extracted (AVE). The reliability of an indicator is evaluated using Cronbach's alpha and composite reliability values. Indicators are considered to be reliable when these values exceed 0.7. According to Table 6, all indicators meet this reliability criterion, ensuring their effectiveness in evaluating the SEM model. AVE is another critical metric used to assess convergent validity, where a value above 0.5 indicates strong convergent validity, a standard met by all indicators as shown in Table 6.

Moreover, cross-loading factor values are essential in the discriminant validity test, which ensures that the values of each latent model are distinct from those of other indicators. Analysis based on cross-loading factor values confirms that each latent model's values are uniquely distinguishable from those of other indicators.

The subsequent analysis involved bootstrapping, as illustrated in Figure 15. This examination tested three hypotheses, concluding that each demonstrated a significant impact. Specifically, Hypothesis 1 (H1) showed that RBL-STEM significantly relates to Conjecturing Thinking Skills (CTS) with a p-value of 0.000 (less than 0.05) and a t-value of 33.267 (greater than 1.96), suggesting a strong association between RBL-STEM and Conjecturing Thinking Skills. Hypothesis 2 (H2) revealed a significant relationship between RBL-STEM and Ethnomathematics (E), with a p-value of 0.000 and a t-value of 33.572, both indicating a significant correlation. Hypothesis 3 (H3) found that Ethnomathematics (E) are significantly associated with Conjecturing Thinking Skills (CTS), evidenced by a p-value of 0.000 and a t-value of 35.859.

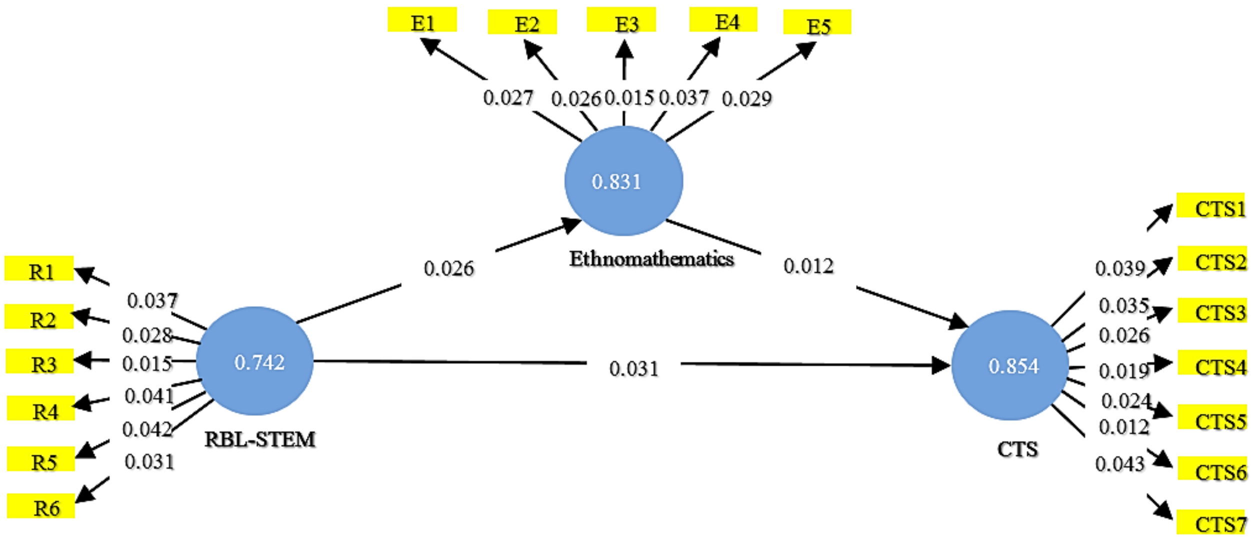


Figure 15. Bootstrapping Analysis

Further analysis was conducted on the direct and indirect effects' path coefficients. The path from RBL-STEM to CTS has a p-value of 0.031; from RBL-STEM to E, the p-value is 0.026; from E to CTS, the p-value is 0.012; and for the indirect path from RBL-STEM through E to CTS, the p-value is 0.0083. These findings indicate that there's a significant effect on the indirect path coefficient, demonstrating that the synergy between RBL-STEM and E significantly affects conjecturing thinking skills.

3-6- Discussion

This study has investigated the impact of RBL-STEM on students' surmising skills, compared to previous studies conducted by several researchers using the same model this study produced a solution that was superior to previous studies conducted by Ibrahim et al. [28], Sutarto et al. [29], and Wardani et al. [30]. The results demonstrate a stark contrast between students' abilities to conceptualize local wisdom-based batik motifs using the (a,d)-edge antimagic coloring technique and geometric transformation concept under the implementation of RBL-STEM between the control and the experiment classes. The observed improvement in students' conjecturing skills under the implementation of RBL-STEM can be attributed to several factors inherent in the RBL-STEM. Firstly, RBL-STEM integrates real-life contexts and hands-on activities, which likely enhanced engagement and facilitated a deeper understanding of complex concepts like the (a,d)-edge antimagic coloring technique and geometric transformations. Such an approach allows students to see the practical applications of abstract mathematical theories, thereby fostering a more intuitive grasp of conjecturing. Secondly, the active learning environment encouraged by RBL-STEM promotes problem-solving and critical thinking skills, essential components related to effective conjecturing. This environment contrasts with traditional teaching methods, possibly used in the control classes, which may focus more on rote learning and less on the exploration of ideas. Finally, the collaborative aspect of RBL-STEM, where students work together to solve problems and develop batik motifs, likely contributed to a richer exchange of ideas, enhancing conjectural thinking through peer interaction and feedback. Thus, the combination of practical application, active learning, and collaboration in RBL-STEM provides a comprehensive environment conducive to the development of conjecturing skills.

This result is in line with DiCerbo (2014), which states that improving students' conjecturing skills is inherently complex due to the multifaceted nature of conjecturing, which involves not only understanding mathematical concepts but also applying them creatively to solve problems [38]. The integration of RBL-STEM is a particularly suitable solution in addressing the complexity inherent in the problem, as it combines Research Based Learning with the STEM approach, thus providing a holistic educational approach. RBL-STEM engages students by connecting mathematical concepts to real-world scenarios, making abstract ideas more accessible and relatable. Meanwhile, Dafik et al. (2023) and Sumardi et al. (2023) also indicated that this combination encourages students to experiment and learn through discovery, which is crucial for developing conjecturing skills that rely heavily on hypothesis formation and testing [7, 17]. Additionally, the collaborative projects typical in RBL-STEM environments foster critical thinking and peer-to-peer interaction, which are essential for refining conjecturing abilities as students challenge and learn from each other's insights. Finally, the STEM component ensures a rigorous analytical framework, guiding students to structure their conjectures logically and empirically test them, thus effectively nurturing a deeper, more comprehensive set of skills necessary for advanced conjecturing.

The enhancement of students' conjecturing thinking skills holds significant practical importance, particularly in fostering the development of critical, logical, and systematic thinking abilities [39]. These skills empower students to construct rational, pattern- or relationship-based arguments, which are instrumental in addressing complex problems. Moreover, conjectural thinking facilitates the improvement of problem-solving abilities through the formulation of initial hypotheses that can be further tested [40]. Furthermore, this skill fosters creativity in the generation of novel and innovative solutions while encouraging effective collaboration in group discussions, such as in research projects or team-based tasks. In research-based learning, this skill plays an instrumental role in supporting scientific understanding, from hypothesis development to conclusion testing. In the world of work, conjectural thinking is applicable to fields that require data analysis, technological innovation, and pattern-based predictions [41]. Conjecturing skills are not only theoretical but also have practical applications in real-world contexts. One of the applications of conjecturing skills can be applied when forecasting using artificial neural networks with timeseries forecasting techniques. Consequently, enhancing these skills has the potential to impact both academic achievement and the preparation of students for the challenges of professional life and everyday decision-making.

Moreover, Nikiforos & Kolyvas (2020) determined that the significant improvement in students' conjecturing skills under the implementation of RBL-STEM as collaborative learning activities can be attributed to several interconnected factors inherent to the RBL-STEM pedagogical approach [42]. First, RBL-STEM actively integrates STEM disciplines with real-world problems and projects, which not only contextualizes learning but also makes mathematical concepts relevant and engaging [7]. This relevance likely enhances student motivation and deepens their understanding of mathematical theories by applying them in meaningful ways. Furthermore, Van der Westhuizen & Bailey (2022) and Alpian et al. (2023) continued that the hands-on and exploratory nature of RBL-STEM encourages active participation and self-directed learning, crucial for developing the critical thinking and analytical skills required for effective conjecturing [5, 43]. In this setting, students are not merely passive recipients of information but are active learners, exploring and testing hypotheses, which is essential for mastering the art of conjecturing. Additionally, Yang & Feng (2022) claimed that the collaborative projects within RBL-STEM promote social learning and intellectual exchange, allowing students to benefit from diverse perspectives and constructive criticism, further refining their conjecturing abilities [34, 44]. Together, these elements create a robust learning environment that significantly fosters and enhances students' ability to conjecture effectively, as evidenced by the observed outcomes in the study. Finally, it can be posited that the implementation of RBL-STEM learning materials can facilitate students' abilities to engage in conjecturing.

4- Conclusion

In conclusion, the research findings indicate that there is a notable disparity in the students' conjecturing thinking skills when developing local wisdom-based batik motifs using the (a,d)-edge anti-magic coloring technique and geometric transformation concept under the implementation of RBL-STEM between the control and experiment classes. The statistical analysis indicates that at the 5% confidence level, the t-statistic of Sig. (2-tailed) = 0.036 is less than 0.05, which implies that the null hypothesis (H0) is rejected. This indicates that the implementation of RBL-STEM learning materials can enhance students' conjecturing abilities, particularly in the development of local wisdom-based batik motifs utilizing the (a,d)-edge antimagic colouring technique, which is further enhanced by the incorporation of geometric transformation concepts.

However, future research could explore a broader application across different educational settings to verify the effectiveness and adaptability of the approach. A longitudinal study would be beneficial to assess the sustainability of these skills over time and their impact on further educational outcomes. Additionally, comparing RBL-STEM with other innovative educational methods could provide deeper insights into its unique benefits and areas for improvement. There is also potential in integrating technology such as augmented or virtual reality to further engage students and enhance the learning experience. Finally, conducting in-depth qualitative research through interviews and observations would provide a richer understanding of student perceptions and the educational impact of RBL-STEM, allowing for refined and more.

5- Declarations

5-1- Author Contributions

Conceptualization, S., A.I.K., and D.; methodology, D., A.F., and M.V.; software, S., M.V., and D.T.; validation, S., A.I.K., and D.T.; formal analysis, S., A.I.K., and D.; investigation, A.I.K., D., and A.F.; resources, S., A.I.K., and D.; data curation, A.I.K., D., and A.F.; writing—original draft preparation, S. and D.; writing—review and editing, S., A.I.K., D., and A.F.; visualization, A.F., M.V., and D.T.; supervision, S., A.I.K., D., A.F., M.V., and D.T.; project administration, S. and A.I.K.; funding acquisition, S, A.I.K., D., and D.T. all authors have read and agreed to the published version of the manuscript.

5-2- Data Availability Statement

The data presented in this study are available on request from the corresponding author.

5-3- Funding and Acknowledgments

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5-4- Institutional Review Board Statement

The study was conducted in accordance with the Social Research Ethics Commission by the institutional review board (or ethics committee) of the Faculty of Education University of Jember (6524/UN25.5.1/LL/2023).

5-5- Informed Consent Statement

Informed consent was obtained from all subjects involved in the study.

5-6- Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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