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Combining a Moving Average with a Triple EWMA Chart to Improve Detection Performance

Piyatida Saesuntia [1], Yupaporn Areepong [1], Saowanit Sukparungsee [1*]

[1] *Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok 10800, Thailand.*

Abstract

This article aims to introduce the novel mixed triple exponentially weighted moving average-moving average (MTEM) chart to accurately detect position changes for both symmetric and non-symmetric distributions. The MTEM chart constructs a moving average (MA) structure to filter out fluctuations in the raw data and then applies triple exponential weighting to improve the ability to identify minor shifts. The average run length (ARL) and median run length (MRL), which are run length profiles derived from the Monte Carlo simulation (MC) strategy, were used to compare the performance of the suggested chart with that of MA, EWMA, TEWMA, and mixed moving average-triple exponentially weighted moving average (MMTE) charts. In addition, the expected average run length (EARL) and expected median run length (EMRL) were also used to rate the overall results. Results of the study indicate that the MTEM chart surpasses competitor charts in detecting minor to moderate changes. The MMTE chart responds slightly slower than the proposed chart. Due to its smoothed and re-averaged structure, it may lose significant information. The MA chart worked better for greater shifts. Furthermore, the MTEM chart competency was applied to two real-world datasets, confirming its practicality.

Keywords:

Combined Control Chart;
Triple Exponentially Weighted Moving Average;
Process Mean;
Tracking;
Average Run Length.

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1- Introduction

Product quality is crucial in industry, as it directly influences brand credibility, economic value, and overall business competitiveness. In order to ensure that products meet client expectations, Statistical Process Control (SPC) is one of the most widely accepted statistical strategies for the monitoring, management, and improvement of manufacturing or operational processes [1]. Among the core tools of SPC are control charts, which are used to track and manage process performance. These charts help determine whether a process remains within control limits or exhibits signs of abnormal variation. The principle of control charts was initially presented by Shewhart in the 1920s [2], which is the Shewhart chart. This chart serves to identify alterations in process means or variability over time and is particularly effective at identifying significant changes in a process. Page [3] tracked the evolution of process inspection procedures from the Shewhart approach to a new chart, which is the cumulative sums (CUMSUM) chart. The CUSUM chart provides the ability to identify minute cumulative modifications in the process, which is beneficial in revealing trends and early signs of abnormalities. In the 1950s, Roberts [4] created the exponentially weighted moving average (EWMA) chart. The EWMA chart gives great significance to recent data while still incorporating past observations, enabling effective tracking of long-term process trends without completely disregarding historical information. Later, Lucas & Saccucci [5] estimated the ARL by Markov chain methodology for the efficacy of both the EWMA and CUSUM schemes. Their findings indicated that the performance characteristics of the EWMA scheme closely resemble those of the CUSUM

[*] **CONTACT**: saowanit.s@sci.kmutnb.ac.th

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scheme. In the 1990s, Shamma & Shamma [6] were the ones who presented the double EWMA (DEWMA) diagram for process tracking. The DEWMA chart's performance was evaluated by comparing it against the Shewhart chart as well as the regular EWMA chart using the ARL characteristic. The proposed DEWMA chart surpasses the Shewhart chart under minor to modest changes, and its ARL is comparable to the standard EWMA chart, whose ARL values are obtained using MC simulation.

The generalized weighted moving average (GWMA) chart was proposed by Sheu & Lin [7], representing an advancement derived from the EWMA chart. This chart demonstrates a higher level of sensitivity than the EWMA chart when it involves spotting small changes in the procedure, as measured by the ARL values. Alemi [8] initially generated the Tukey control plot to estimate confidence intervals for medians, independent of normal distribution. It is steady and unaffected by sporadic anomalous findings, and it can be used with tiny data sets. Khoo [9] proposed the MA control chart to quickly discover small-scale variations in non-conforming levels. The EWMA and DEWMA schemes were tested for robustness under normality conditions by Alkahtani [10]. The DEWMA scheme performs significantly better than the EWMA scheme. Khan et al. [11] established a chart known as the modified EWMA (MEWMA), and its efficacy was compared to that of the traditional MEWMA and EWMA charts. It revealed that the suggested chart is able to rapidly spot small shifts. According to the statistical analysis, in the 2021s, Alevizakos et al. [12] conducted a study of the TEWMA chart. The TEWMA chart detects minor changes better than EWMA, DEWMA, and GWMA, as indicated by the run-length properties analysis. Alevizakos et al. [13] suggested MEWMA and double MEWMA (DMEWMA) techniques to improve change detection utilizing ARL and standard deviation of run length (SDRL) values from MC method to assess their effectiveness. The results demonstrated that both MEWMA and DMEWMA techniques surpass the EWMA and double EWMA (DEWMA) techniques in the detection of process changes.

Since the single charts often have limitations in detecting small changes, especially when process variability increases or noise is high, and because they assume a normal data distribution, some charts may not perform well in cases of skewed or non-normal distributions. Many scholars have suggested the combination chart as a strategy that combines the advantages of each chart and reduces its limitations. By combining different statistical structures, the combination chart can detect small changes more quickly, be robust to changes in the distribution of the data, and reduce the impact of high variance. For instance, Zaman et al. [14] produced a chart known as the CUSUM-EWMA. The EWMA-MA method was first introduced by Khan et al. [15]. This chart is intended to generate high-performance control charts for processes in which the quality variables exhibit an exponential spread by integrating the stability of MA and EWMA. The mixed nonparametric Tukey MA-DEWMA chart was devised by Taboran et al. [16]. Tukey MA-DEWMA chart is quality control instrument that assist in identifying changes in average for production, whether the data is balanced or unbalanced. In the 2022s, Mahmood et al. [17] introduced the TEWMA-Tukey method.

Talordphop et al. [18, 19] proposed a variety of mixed MEWMA-MA and Tukey EWMA-MEWMA schemes to monitor changes. These charts demonstrated superior performance in detecting small changes compared to their single-chart counterparts. Additionally, Sukparungsee & Phantu [20] used an explicit formula to analyze the average time to signal (ATS) for the MA chart with the ZMGINAR(1) form in the 2024s. The results indicated that the MA diagram outperformed the Shewhart diagram in all the evaluated scenarios. Later, Tyagi & Yadav [21] proposed a hybrid quality control scheme between EWMA and MEC by applying an autoregressive procedure to the auto-correlated observations and applying chart construction techniques directly to the residuals. The ARL is used as an effectiveness indicator to assess the impact of the scheme and compare its performance with MEC, CUSUM, and EWMA methods. The outcomes designate that this method is more delicate in detecting slight to moderate changes than the compared charts. The nonparametric EWMA-MA (NPEWMA-MA) based on signed-rank and sign statistic was introduced by Raza et al. [22, 23]. Both charts were developed to track shifts in the proposed process location. The chart performance was evaluated using the MC method with ARL, SDRL, and MRL metrics. The NPEWMA-MA, based on signed-rank and sign statistics, demonstrated greater performance compared to existing nonparametric charts.

In the 2025s, the nonparametric MEWMA based on sign rank for zero-inflated data was presented by Phantu et al. [24]. The MEWMA-SR performance visibly shows that the nonparametric MEWMA-SR chart is more active at identifying shifts in the likelihood of small-measure change. Overall, numerous researchers have confirmed that mixed or combined control charts consistently outperform individual charts in detecting process abnormalities.

However, the efficacy of TEWMA and MA control charts decreases as the variance increases when the data are non-normal, despite their effectiveness in detecting small mean changes or in the early stages after process changes. No integrated approach can simultaneously address these limitations. Which served as the impetus for the creation of the proposed TEWMA-MA chart in order to simultaneously mitigate the effects of variance, increase robustness to non-normal distributions, and speed up detection. In this study, we propose the mixed TEWMA-MA, or MTEM, which combines the smoothing and transformation sensitivity of TEWMA with the noise reduction capabilities of MA. The performance evaluation utilized the run-length characteristics and compared them with MA, EWMA, TEWMA, and MMTE charts under both symmetric and asymmetric distributions. To validate its practical effectiveness, the MTEM chart is also applied to two real-world datasets involving carbon dioxide (CO_2) emissions from natural gas sources in Thailand and the remission times of bladder cancer patients. To validate its practical effectiveness, the MTEM chart is also applied to two real-world datasets involving carbon dioxide (CO_2) emissions from natural gas sources in Thailand and the duration of remission for people with bladder cancer.

2- Control Charts

The structures of the MA, EWMA, TEWMA, MMTE, and MTEM charts are explained in this part such as the calculation principles of statistics, mean, variance and control limits. Let X_i, $i = 1, 2, \ldots$, designate independent random samples drawn from a normal distribution.

2-1- MA Chart

The MA chart was first shown by Khoo [9] in 2004s. The MA chart's fundamental concept is to use the average of a number of past data to reduce the effect of short-term fluctuations. The essential advantage of the MA chart is its ability to effectively reduce data noise, making it well-suited for detecting gradual or cumulative changes in a process. At i time, the MA (MA_i) statistics is:

$$MA_i = \begin{cases} \frac{X_i+X_{i-1}+X_{i-2}+\cdots}{i}, & i < w \\ \frac{X_i+X_{i-1}+\ldots+X_{i-w+1}}{w}, & i \geq w \end{cases} \tag{1}$$

where w is window size. The expected value and variance of the MA statistics stand delineated in Equations 2 and 3.

$$E(MA_i) = \mu_0 \tag{2}$$

and

$$Var(MA_i) = \begin{cases} \frac{\sigma^2}{i}, & i < w \\ \frac{\sigma^2}{w}, & i \geq w \end{cases} \tag{3}$$

Consequently, the following formula can be used to determine the inner boundary (UCL) and outer boundary (LCL).

$$UCL/LCL = \begin{cases} \mu_0 \pm \frac{L_1\sigma}{\sqrt{i}}, & i < w \\ \mu_0 \pm \frac{L_1\sigma}{w}, & i \geq w \end{cases} \tag{4}$$

where L_1 denotes the control limit factor used in the MA chart. The mean of the in-control process is represented by μ_0, while the standard deviation of the process is denoted by σ.

2-2- EWMA Chart

Roberts [4] was the first person to present the EWMA chart. EWMA is utilized for the purpose of monitoring the process's mean by placing greater emphasis on the most recent data rather than historical data. It is particularly well-suited for detection of minor shifts in the mean. EWMA statistic (E_i) is:

$$E_i = \lambda X_i + (1-\lambda)E_{i-1} \tag{5}$$

where λ is a parameter for weighting, with a value greater than 0 and less than or equal to 1. The value that is initially established is $E_0 = \mu_0$. The EWMA statistic's mean and variance are calculated using the following formulas:

$$E(E_i) = \mu_0 \tag{6}$$

$$Var(E_i) = \sigma^2\left(\frac{\lambda}{2-\lambda}\right) \tag{7}$$

The inner and outer boundaries for the EWMA chart are given by the formulas presented below:

$$UCL = \mu_0 + L_2\sigma\sqrt{\frac{\lambda}{2-\lambda}} \tag{8}$$

$$LCL = \mu_0 - L_2\sigma\sqrt{\frac{\lambda}{2-\lambda}} \tag{9}$$

where L_2 serves as the control limit factor associated with EWMA chart. μ_0 is the mean of the procedure under in-control condition. σ refers to standard deviation.

2-3- TEWMA Chart

The TEWMA chart was invented by Alevizakos et al. [12], which is an extension to both the EWMA and DEWMA charts. A more sensitive detection of minute changes to the process mean is its principal goal. The three-layer exponential weighting smoothing technique is the defining characteristic of the TEWMA chart. This structure allows the TEWMA chart to detect subtle process changes more rapidly and effectively as compared to EWMA and DEWMA methods, especially with small shifts. A description of the equation system employed for the TEWMA statistics can be seen below (T_i).

$$E_i = \lambda X_i + (1-\lambda)E_{i-1}$$
$$D_i = \lambda E_i + (1-\lambda)D_{i-1} \tag{10}$$
$$T_i = \lambda D_i + (1-\lambda)T_{i-1}$$

where E_i, D_i, and T_i are the statistics for EWMA, DEWMA, and TEWMA, respectively. The initial values are denoted by $E_0 = D_0 = T_0 = \mu_0$. The weighting parameter is denoted by $(0 < \lambda \leq 1)$. By applying the formula provided below, it is feasible to ascertain the variance and anticipated values of the TEWMA chart.

$$E(T_i) = \mu_0 \tag{11}$$

$$Var(T_i) = \left[\frac{6\lambda(1-\lambda)^6}{(2-\lambda)^5} + \frac{12\lambda^2(1-\lambda)^4}{(2-\lambda)^4} + \frac{7\lambda^3(1-\lambda)^2}{(2-\lambda)^3} + \frac{\lambda^4}{(2-\lambda)^2}\right]\sigma^2 \tag{12}$$

In the TEWMA control chart, the UCL and LCL are as follows:

$$UCL/LCL = \mu_0 \pm L_3\sigma\sqrt{\frac{6\lambda(1-\lambda)^6}{(2-\lambda)^5} + \frac{12\lambda^2(1-\lambda)^4}{(2-\lambda)^4} + \frac{7\lambda^3(1-\lambda)^2}{(2-\lambda)^3} + \frac{\lambda^4}{(2-\lambda)^2}} \tag{13}$$

where L_3 serves as the control limit factor associated with the TEWMA chart. The mean of the procedure under in-control condition is represented by μ_0, while the standard deviation is denoted by σ.

2-4- MA-TEWMA (MMTE) Chart

The MMTE combination control chart effectively attains the desired performance by integrating the MA scheme with the TEWMA scheme. The values obtained from TEWMA (triple weighted smoothing) are used to create a moving average to reduce the volatility of the TEWMA statistic. Then, the MMTE statistic (MT_i) as following:

$$MT_i = \begin{cases} \frac{T_i + T_{i-1} + T_{i-2} + \cdots}{i}, & i < w \\ \frac{T_i + T_{i-1} + \ldots + T_{i-w+1}}{w}, & i \geq w \end{cases} \tag{14}$$

where T_i denotes its TEWMA statistic at time *i*. Consequently, the boundaries for the MTEM chart can be created in the manner that is described below.

$$UCL/LCL = \begin{cases} \mu_T \pm L_4\sigma_T\sqrt{\left[\frac{6\lambda(1-\lambda)^6}{(2-\lambda)^5} + \frac{12\lambda^2(1-\lambda)^4}{(2-\lambda)^4} + \frac{7\lambda^3(1-\lambda)^2}{(2-\lambda)^3} + \frac{\lambda^4}{(2-\lambda)^2}\right]\frac{1}{i}}, & i < w \\ \mu_T \pm L_4\sigma_T\sqrt{\left[\frac{6\lambda(1-\lambda)^6}{(2-\lambda)^5} + \frac{12\lambda^2(1-\lambda)^4}{(2-\lambda)^4} + \frac{7\lambda^3(1-\lambda)^2}{(2-\lambda)^3} + \frac{\lambda^4}{(2-\lambda)^2}\right]\frac{1}{w}}, & i \geq w \end{cases} \tag{15}$$

where L_4 refers to the control limit factor associated with the MMTE chart. The mean of the TEWMA chart denotes μ_T, while its standard deviation is represented by σ_T. λ is the weighting parameter $(0 < \lambda \leq 1)$.

2-5- TEWMA-MA (MTEM) Chart

Similarly, the MTEM strategy is a new hybrid chart that is developed by combining the advantages of both the TEWMA and MA charts. The fundamental concept of MTEM chart is to substitute the observed values (x) in the

TEWMA chart's statistical formulation with the MA statistics (MA_i) to reduce the noise of the raw data before smoothing the data three times to increase the sensitivity of detecting changes in the smoothed data. The resulting MTEM statistics (TM_i) are defined as follows:

$$EM_i = \lambda MA_i + (1-\lambda)EM_{i-1}$$
$$DM_i = \lambda EM_i + (1-\lambda)DM_{i-1} \quad (16)$$
$$TM_i = \lambda DM_i + (1-\lambda)TM_{i-1}$$

where EM_i, DM_i, and TM_i indicate the EWMA-MA, DEWMA-MA, and TEWMA-MA statistics, respectively. The EWMA-MA, DEWMA-MA, and TEWMA-MA statistic's initial values are determined by a fixed initialization method that is equivalent to the process target or in-control mean (μ_0), so $EM_0 = DM_0 = TM_0 = \mu_0$. λ is the symbol for the weighting parameter, with a range of 0 to 1. The following are the boundaries of control.

$$UCL/LCL = \mu_{MA} \pm L_5\sigma_{MA}\sqrt{\left[\frac{6\lambda(1-\lambda)^6}{(2-\lambda)^5} + \frac{12\lambda^2(1-\lambda)^4}{(2-\lambda)^4} + \frac{7\lambda^3(1-\lambda)^2}{(2-\lambda)^3} + \frac{\lambda^4}{(2-\lambda)^2}\right]\frac{1}{w}} \quad (17)$$

where L_5 denotes the control limit constant associated with the MTEM chart. The symbol μ_{MA} means the average of the MA chart, while its standard deviation is represented by σ_{MA}.

3- Methodology of Performance Measurement

An effective control chart is characterized by its capability to promptly notice shifts in the procedure. Two commonly used statistical measures for evaluating this performance are the ARL and MRL. ARL mentions to the mean quantity of observations taken before the chart signals an alarm. When the procedure is under control, The value of the ARL_0 has to be high, indicating a low alarm frequency. In an out-of-control procedure, the ARL_1 is expected to be low, demonstrating the chart's effectiveness in quickly identifying process shifts. The median run length, also known as MRL, is the mean quantity of data points that need to be analyzed once an alarm has been triggered. The ARL and MRL values are obtained by the MC method and can be calculated as follows:

$$ARL = \frac{\sum_{t=1}^{M} RL_t}{M} \quad (18)$$

and;

$$MRL = Median(RL) \quad (19)$$

where RL_t represents the amount of samples that were evaluated prior to the graph signals that the procedure is out of control at iteration *t*. *M* represents the amount of iterations established at 100,000 to ensure greater accuracy in estimating the run length profile, in accordance with the law of large numbers. Increasing the number of repetitions also reduces the standard error for the run length estimates.

Furthermore, the EARL and EMRL were introduced by You et al. [25, 26]. The EARL is a performance metric used to assess how effectively a control chart detects process changes across various levels of parameter shifts. The EMRL shows the average number of samples (or cycles of sampling) needed before the chart the chart implies the procedure is out of control, taking into account different sizes of changes. Both EARL and EMRL provide a comprehensive view of a control chart's overall performance, rather than focusing on a single change point. The EARL and EMRL can be estimated using Equations 20 and 21, as shown below.

$$EARL = \frac{1}{\delta_2-\delta_1}\int_{\delta_1}^{\delta_2} ARL(\delta)d\delta \quad (20)$$

and;

$$EMRL = \frac{1}{\delta_2-\delta_1}\int_{\delta_1}^{\delta_2} MRL(\delta)d\delta \quad (21)$$

where δ_1 represents the least significant shift, while δ_2 represents the most significant shift. Figure 1 illustrates the run-length profile calculation procedure.

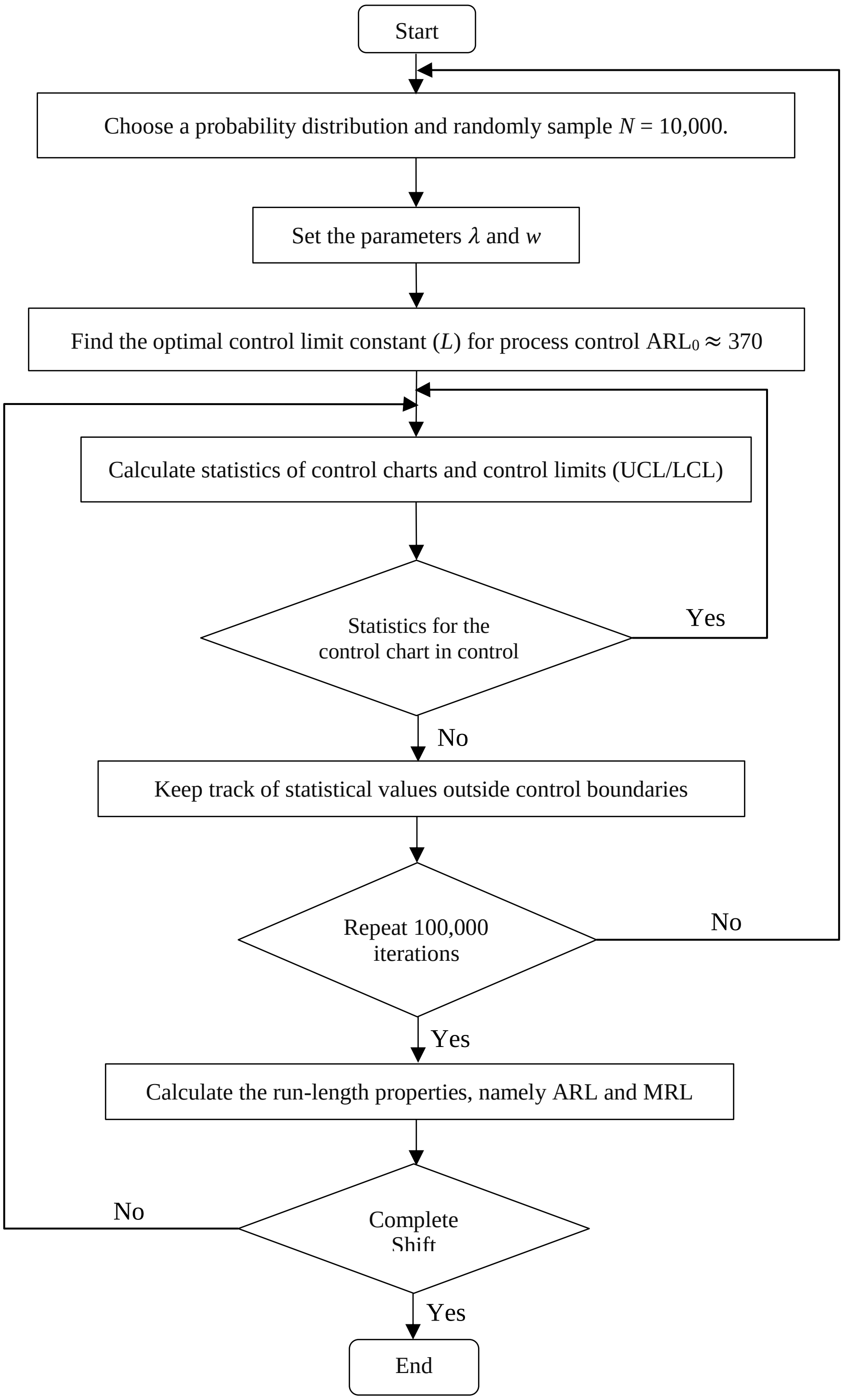


Figure 1. The calculation process of the run-length properties

4- Simulation Results

The primary goal of investigation aims to estimate the efficacy of the suggested MTEM chart to spot shifts in the procedures mean. The MA, EWMA, TEWMA, and mixed MMTE charts are all compared to this one. The efficiency evaluation focuses on the ARL and MRL values obtained through MC method under symmetric and non-symmetric. The normal $N(0,1)$ and Laplace $L(0,1)$ distributions are considered as symmetric distribution under study, whereas exponential Exp(5) and gamma $\Gamma(4,1)$ distributions are considered as asymmetric distribution under study. In controlled procedures, a high value of ARL_0 is required to indicate a low rate of wrong alarms. Conversely, when the procedure is out of control, the ARL_1 should be low, as it reflects the chart's skill to rapidly detect process shifts. For in-control processes, the control chart statistics remain within the LCL and UCL. On the other hand, when the process is not under controlled conditions, the statistics exceed the UCL or fall below the LCL. In this study, ARL_0 is fixed at 370. The shift size (δ) of the process for the symmetric distribution is evaluated positively and negatively at values of ± 0.05, ± 0.10, ± 0.25, ± 0.50, ± 0.75, ± 1.00, ± 1.50, ± 2.00, and ± 3.00. For the asymmetric distribution, it is evaluated positively at values of 0.05, 0.10, 0.25, 0.50, 0.75, 1.00,1.50, 2.00, and 3.00.

The estimations of the ARL and MRL for the presented MTEM chart are presented in Table 1. ARL_0 value is approximately 370, λ = 0.25 and varying window sizes (w) of 2, 5 and 10. The normal (0, 1) distribution is the studied symmetric distribution, The results show that as the w increases, both ARL_1 and MRL values decrease, while the control boundary constant of the MTEM chart (L_5) rises. The exponential (5) distribution has been considered according to a non-symmetric distribution, the results are consistent with the normally distributed (0,1) observations, as illustrated in Table 2.

Table 1. **The run-length profile estimation of the suggested MTEM chart under normal distribution with varying parameter w, $ARL_0 \approx 370$, and $\lambda = 0.25$**

Shifts (δ)	w = 2		w = 5		w = 10	
	L_5 = 3.42675		L_5 = 5.24		L_5 = 6.8125	
	ARL	MRL	ARL	MRL	ARL	MRL
-3.00	3.81786	4	3.75596	4	3.55073	3
-2.00	5.13367	5	5.08361	5	4.81928	5
-1.50	6.47505	6	6.39977	6	6.10677	6
-1.00	9.75810	9	9.48039	9	9.05135	8
-0.75	14.38274	12	13.79996	12	12.93348	11
-0.50	28.22822	22	26.96522	21	24.75535	19
-0.25	93.15392	67	90.38412	64	84.06416	58
-0.10	254.7715	178	252.2456	175	244.5379	165
-0.05	332.3383	231	332.3207	229	328.9915	221
0	370.2070	258	370.6470	256	370.2872	249
0.05	332.9051	232	332.3265	230	328.4558	221
0.10	254.2753	179	252.1738	176	245.2320	166
0.25	93.04003	67	90.46771	64	83.86038	58
0.50	28.11411	22	26.83020	20	24.55571	19
0.75	14.32545	12	13.71331	12	12.83873	10
1.00	9.71286	9	9.43072	8	8.99712	8
1.50	6.45891	6	6.37475	6	6.08511	6
2.00	5.12182	5	5.06754	5	4.80405	5
3.00	3.80965	4	3.74843	4	3.54274	3

Table 2. **The run-length properties estimation of the suggested MTEM chart under exponential distribution with varying parameter w, $ARL_0 \approx 370$, and $\lambda = 0.25$**

Shifts (δ)	w = 2		w = 5		w = 10	
	L_5 = 5.925875		L_5 = 9.36365		L_5 = 13.2245	
	ARL	MRL	ARL	MRL	ARL	MRL
0	370.8518	267	370.9946	267	370.7477	266
0.05	259.8800	190	258.4481	189	254.0599	186
0.10	192.4142	143	189.6953	141	184.6411	137
0.25	98.17456	77	96.32115	75	91.92455	72
0.50	53.69759	46	52.41813	45	49.52888	42
0.75	39.99795	36	39.09919	35	37.07433	33
1.00	34.25002	32	33.54682	31	32.10187	30
1.50	29.68692	29	29.26015	29	28.42061	28
2.00	27.94529	28	27.64109	27	27.05238	27
3.00	26.50200	26	26.28951	26	25.86464	26

The analysis was performed using various weighting parameter (λ) values of 0.1, 0.25 and 0.5 with $ARL_0 \approx 370$, w = 0.25 under symmetric and asymmetric distributions. The normal (0, 1) distribution is studied as symmetric, the results are presented in Tables 3. The studied non-symmetric distribution is an exponential (5) distribution, the results are presented in Tables 4. The results for both normal (0, 1) and exponential (5) distributions are similar, showing that the value of λ went up, MTEM chart control limit constant (L_5) increased, while ARL_1 decreased.

Table 3. The run-length properties estimation of the suggested MTEM chart under normal distribution with varying parameter λ, $ARL_0 \approx 370$, and w = 5

Shifts (δ)	λ = 0.10		λ = 0.25		λ = 0.50	
	L_5 = 4.4375		L_5 = 5.24		L_5 = 5.245	
	ARL	MRL	ARL	MRL	ARL	MRL
-3.00	8.73648	9	3.75596	4	1.43004	1
-2.00	10.88186	11	5.08361	5	2.39242	2
-1.50	12.82469	13	6.39977	6	3.58019	3
-1.00	16.51058	16	9.48039	9	7.37136	6
-0.75	20.31406	19	13.79996	12	13.68304	10
-0.50	29.55019	26	26.96522	21	34.05442	24
-0.25	71.95390	56	90.38412	64	122.5491	84
-0.10	212.0744	152	252.2456	175	285.4759	194
-0.05	312.3662	220	332.3207	229	345.0882	235
0	370.2029	261	370.6470	256	370.5812	252
0.05	312.2219	221	332.3265	230	344.9865	234
0.10	212.1770	153	252.1738	176	285.1381	194
0.25	71.79376	55	90.46771	64	122.6240	83
0.50	29.40808	26	26.83020	20	33.77511	23
0.75	20.23323	19	13.71331	12	13.64839	10
1.00	16.46344	16	9.43072	8	7.32351	5
1.50	12.80119	13	6.37475	6	3.55015	3
2.00	10.86462	11	5.06754	5	2.38071	2
3.00	8.72611	9	3.74843	4	1.42440	1

Table 4. The run-length properties estimation of the suggested MTEM chart under exponential (5) distribution with varying parameter λ, $ARL_0 \approx 370$, and w = 5

Shifts (δ)	λ = 0.10		λ = 0.25		λ = 0.50	
	L_5 = 15.3905		L_5 = 9.36365		L_5 = 5.967153	
	ARL	MRL	ARL	MRL	ARL	MRL
0	370.1394	284	370.9946	267	370.8868	261
0.05	252.4564	201	258.4481	189	277.7826	196
0.10	190.0792	157	189.6953	141	214.5295	153
0.25	117.4942	105	96.32115	75	112.4746	81
0.50	88.16947	85	52.41813	45	52.99668	40
0.75	79.79813	79	39.09919	35	32.84576	26
1.00	76.07991	76	33.54682	31	23.88098	20
1.50	72.41277	72	29.26015	29	16.57028	14
2.00	70.50569	70	27.64109	27	13.79944	12
3.00	68.49321	68	26.28951	26	11.782796	11

Table 5 presents the observations for data following a normal distribution $N(0,1)$. When the shift measure (δ) is ±0.05, ±0.10, ±0.25, ±0.50, and ±0.75, the MTEM chart at L_5 = 5.24 exhibits lower ARL_1 and MRL values than competitor charts. At $\delta = \pm 1.00$, the EWMA chart beats others, while the MA chart performs best at $\delta = \pm 1.50$, ± 2.00, and ± 3.00. The MTEM chart responds better to all levels of change compared to TEWMA and MMTE charts. The Laplace distribution results are presented in Table 6. The MTEM chart exhibits the lowest ARL_1 and MRL values at L_5 = 5.3655 over a range of shifts from -1.50 to 1.50. The MA control chart outperforms the other chart at δ = ±2.00 and ±3.00. The chart that has been proposed outperforms EWMA in the range of changes from -1.50 to 1.50, and it outperforms TEWMA in all changes.

Table 5. Run-length properties comparison of the MA, EWMA, TEWMA, MMTE, and MTEM charts for normal distribution when provided $ARL_0 \approx 370$, $\lambda = 0.25$, and $w = 5$

Shifts (δ)	MA		EWMA		TEWMA		MMTE		MTEM	
	$L_1 = 2.885$		$L_2 = 2.9$		$L_3 = 2.44$		$L_4 = 5.215$		$L_5 = 5.24$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
-3.00	**0.54862**	**0**	1.19154	1	3.83401	4	5.58768	6	3.75596	4
-2.00	**1.78138**	**2**	2.4753	2	5.14978	5	6.94897	7	5.08361	5
-1.50	**3.64896**	**3**	4.19553	4	6.501933	6	8.29502	8	6.39977	6
-1.00	10.00938	7	**9.2694**	7	9.84465	9	11.5027	11	9.48039	9
-0.75	20.44531	15	16.97791	13	14.54574	12	16.03251	14	**13.79996**	12
-0.50	51.71274	36	40.3718	29	28.49936	22	29.61445	23	**26.96522**	**21**
-0.25	161.859	113	134.6398	94	93.65766	67	93.60734	68	**90.38412**	**64**
-0.10	311.1153	215	293.4015	204	255.5248	179	269.0378	179	**252.2456**	**175**
-0.05	354.8035	246	347.5777	242	332.9135	233	333.1749	233	**332.3207**	**229**
0	370.8666	257	370.6545	258	370.8578	259	370.584	259	370.647	256
0.05	354.0057	245	348.6896	242	333.6272	233	333.5305	234	**332.3265**	**230**
0.10	310.9596	215	294.2815	203	255.2513	179	254.2741	179	**252.1738**	**176**
0.25	162.8145	113	94.24740	94	93.51251	67	93.43398	68	**90.46771**	**64**
0.50	51.27486	36	40.22039	29	28.42279	22	29.53374	23	**26.8302**	**20**
0.75	20.38526	14	16.94736	13	14.48797	12	15.98088	14	**13.71331**	12
1.00	9.99296	7	**9.23852**	7	9.80268	9	11.46133	11	9.43072	8
1.50	**3.61229**	**3**	4.16966	4	6.49528	6	8.27865	8	6.37475	6
2.00	**1.76557**	**1**	2.45974	2	5.14009	5	6.93843	7	5.06754	5
3.00	**0.54321**	**0**	1.18623	1	3.82746	4	5.58022	6	3.74843	4

Note: The lowest ARL_1 and MRL values are displayed in bold letters.

Table 6. Run-length properties comparison of the MA, EWMA, TEWMA, MMTE, and MTEM charts for Laplace distribution when provided $ARL_0 \approx 370$, $\lambda = 0.25$, and $w = 5$

Shifts (δ)	MA		EWMA		TEWMA		MMTE		MTEM	
	$L_1 = 3.114$		$L_2 = 3.3355$		$L_3 = 2.495$		$L_4 = 5.3345$		$L_5 = 5.3655$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
-3.00	**1.83314**	**2**	2.82529	3	5.00760	5	6.80471	7	4.93817	5
-2.00	**5.30544**	**4**	6.62075	6	6.98199	7	8.75696	8	6.84638	6
-1.50	12.50168	9	13.35480	11	9.36255	9	11.0519	10	**9.06288**	**8**
-1.00	38.61708	27	36.71266	27	16.45632	14	17.86359	15	**15.64569**	**13**
-0.75	74.22837	51	69.53409	49	27.28979	21	28.35095	22	**25.86169**	**20**
-0.50	147.0594	102	139.8096	97	58.43084	43	58.56895	43	**55.67087**	**40**
-0.25	276.7369	192	271.8085	190	169.0072	119	166.6275	118	**164.0644**	**115**
-0.10	353.1996	245	352.2393	247	314.9193	221	313.4747	221	**311.9154**	**216**
-0.05	365.8404	254	366.1588	256	354.7822	249	354.6310	250	**353.9599**	**245**
0	369.9806	257	370.9531	260	370.3992	261	370.7936	261	370.3478	256
0.05	366.9405	255	365.6279	255	355.0765	249	354.6880	249	**354.3405**	**245**
0.10	352.5292	245	350.9380	245	314.6778	220	313.2292	220	**311.1972**	**215**
0.25	278.0553	191	272.4021	189	168.2622	119	166.4685	118	**163.4865**	**114**
0.50	147.0696	102	140.5215	98	58.38971	43	58.41060	43	**55.48748**	**40**
0.75	74.07268	51	69.07748	49	27.25236	21	28.29584	22	**25.79848**	**20**
1.00	38.40226	27	36.57511	27	16.41870	14	17.82392	15	**15.57169**	**13**
1.50	12.45567	9	13.32313	11	9.33502	9	11.01981	10	**9.02257**	**8**
2.00	**5.27714**	**4**	6.59721	6	6.96364	7	8.74077	8	6.81998	6
3.00	**1.83314**	**2**	2.81033	3	4.99575	5	6.79365	7	4.92111	5

Note: The lowest ARL1 and MRL values are displayed in bold letters.

Table 7 demonstrates the shift parameter (δ) change from 0.05 to 0.75, the proposed MTEM chart at L_5 = 9.36365 has lower ARL_1 and MRL values than existing charts when the observations follow exponential (5) distribution. The EWMA chart outperforms the other charts at δ = 1.00. The best scenarios for the MA control chart are δ = 1.50, 2.00, and 3.00. The MTEM chart exhibits a superior response to variations at all levels when contrasted with the TEWMA and MMTE charts. Table 8 displays the observed values that comply with the gamma (4,1) distribution. In comparison with competitor charts, the MTEM control chart at L_5 = 5.3655 provides lower ARL_1 and MRL values when δ = 0.05, 0.10, and 0.25. Over a range of changes from 0.50 to 3.00, the MA control chart works well. The MTEM chart outperforms TEWMA in the range of changes from 0.05 to 0.50, and it outperforms MMTE in all changes. While the EWMA chart outperforms the chart that has been proposed in the range of changes from 0.50 to 3.00.

Table 7. Run-length properties comparison of the MA, EWMA, TEWMA, MMTE, and MTEM charts for exponential distribution when provided $ARL_0 \approx 370$, $\lambda = 0.25$, and $w = 5$

Shifts (δ)	MA		EWMA		TEWMA		MMTE		MTEM	
	L_1 = 2.217968		L_2 = 2.609635		L_3 = 4.1909		L_4 = 9.36345		L_5 = 9.36365	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
0	370.7947	258	370.4864	266	370.8671	267	370.4334	267	370.9946	267
0.05	308.6839	214	279.2233	200	260.4868	190	259.0250	190	**258.4481**	**189**
0.10	259.4048	180	216.6354	157	192.8311	144	191.0357	143	**189.6953**	**141**
0.25	163.1492	114	117.8511	88	98.60884	77	98.07500	77	**96.32115**	**75**
0.50	87.84709	62	60.74227	48	53.99115	46	54.52767	47	**52.41813**	**45**
0.75	53.92592	38	41.09720	35	40.22882	36	41.25826	37	**39.09919**	35
1.00	36.30275	**26**	**32.17853**	28	34.43797	32	35.77631	33	33.54682	31
1.50	**20.15068**	**15**	24.59337	23	29.79675	29	31.46486	31	29.26015	29
2.00	**13.38375**	**10**	21.40741	20	28.02543	28	29.84761	30	27.64109	27
3.00	**7.88743**	**6**	18.67138	18	26.55746	26	28.49667	28	26.28951	26

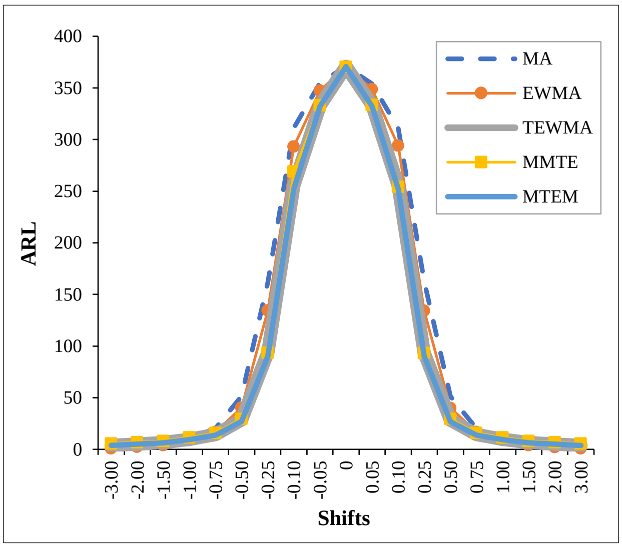
Note: The lowest ARL1 and MRL values are displayed in bold letters

Table 8. Run-length properties comparison of the MA, EWMA, TEWMA, MMTE, and MTEM charts for gamma distribution when provided $ARL_0 \approx 370$, $\lambda = 0.25$, and $w = 5$

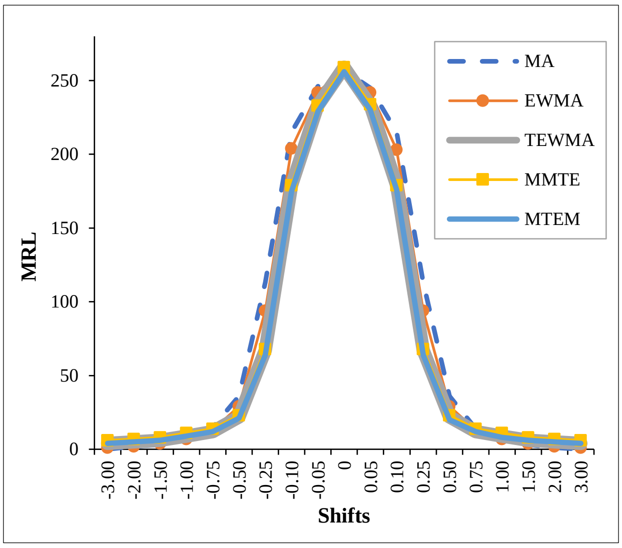
Shifts (δ)	MA		EWMA		TEWMA		MMTE		MTEM	
	L_1 = 3.022		L_2 = 3.179		L_3 = 2.436		L_4 = 5.2075		L_5 = 5.2328	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
0	370.9228	257	370.7030	259	370.4846	261	370.4287	261	370.2570	258
0.05	192.7876	133	188.8111	132	190.6269	134	192.1207	136	**188.6509**	**131**
0.10	109.4301	76	104.0587	73	98.41103	70	99.72454	72	**96.0328**	**68**
0.25	31.29879	22	29.87242	21	27.10328	21	28.65209	22	**26.00409**	**20**
0.50	**9.08512**	7	9.47041	7	10.69576	9	12.39347	11	10.47943	9
0.75	**4.330**70	**3**	4.95516	4	7.06575	6	8.82445	8	7.07199	6
1.00	**2.55182**	**2**	3.18921	3	5.52641	5	7.30411	7	5.60699	5
1.50	**1.21353**	1	1.72782	1	4.07341	4	5.82083	6	4.16198	4
2.00	**0.702**70	**0**	1.10983	1	3.32832	3	5.01269	5	3.39742	3
3.00	**0.30807**	0	0.57070	0	2.51863	2	4.03992	4	2.56004	2

Note: The lowest ARL1 and MRL values are displayed in bold letters.

Figures 2 to 5 illustrates the MA, EWMA, TEWMA, MMTE, and MTEM charts, which present the ARL and MRL graphs under the normal $N(0,1)$, Laplace $L(0,1)$, exponential Exp(5), and gamma Γ(4,1) distributions, respectively. In addition, the EARL and EMRL values are also analyzed across a range of shift magnitudes. This study considers both symmetric and non-symmetric distributions, as shown in Table 9. The suggested control chart always shows lower EARL and EMRL values than the competitor charts for all distributions when the magnitude of change (δ) is in the range of 0.05 to 1.00, showing they are better at quickly spotting small to modest changes in the procedure. Conversely, the MA chart consistently shows the lowest EARL and EMRL values when the change magnitude (δ) is in the range of 1.50 to 3.00, indicating a superior ability to detect large changes.

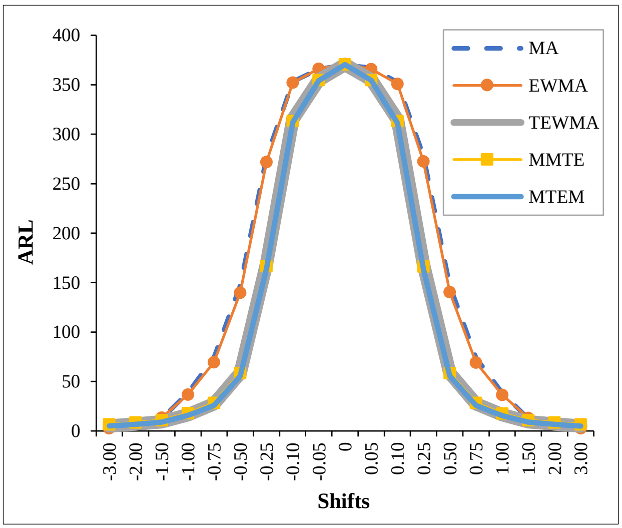


(a)

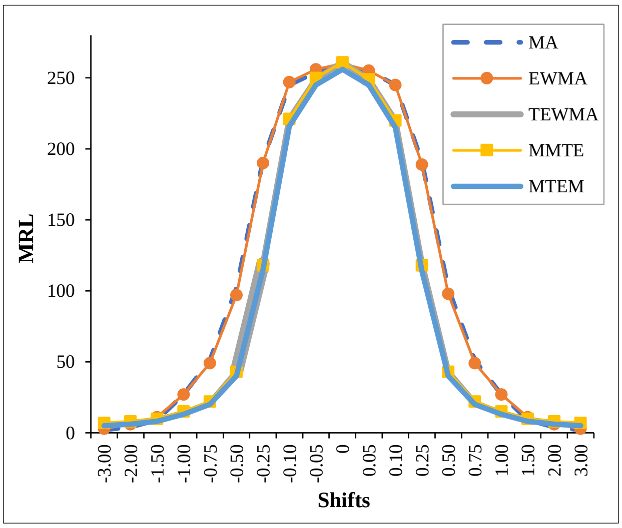


(b)

Figure 2. **Plotting of analysis results for a control chart following normal (0, 1) distribution: (a) ARL values and (b) MRL values**

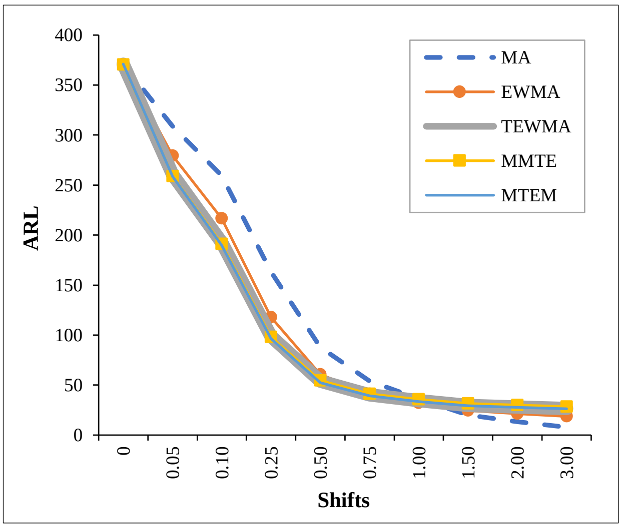


(a)

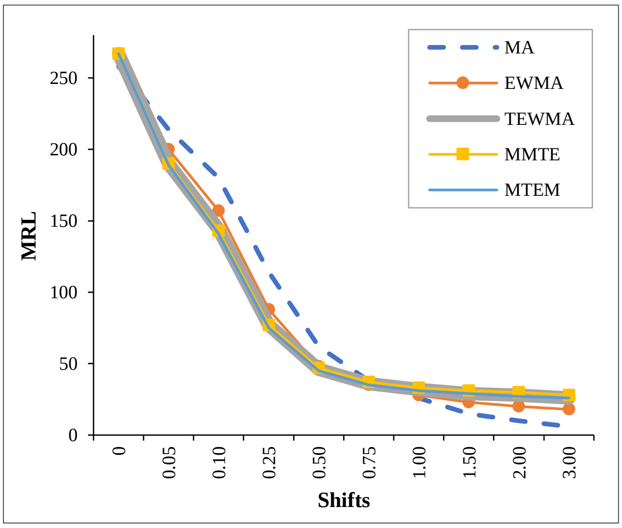


(b)

Figure 3. **Plotting of analysis results for a control chart following Laplace (0, 1) distribution: (a) ARL values and (b) MRL values**

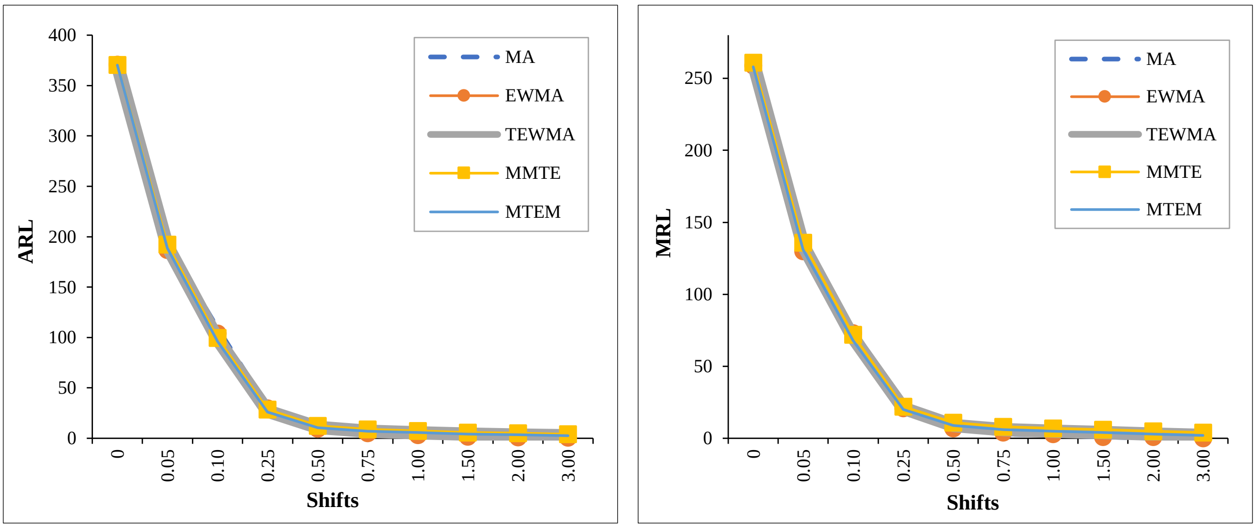


(a)



(b)

Figure 4. **Plotting of analysis results for a control chart following exponential (5) distribution: (a) ARL values and (b) MRL values**



(a) (b)

Figure 5. Plotting of analysis results for a control chart following gamma (4, 1) distribution: (a) ARL values and (b) MRL values

Table 9. Display of the EARL and EMRL values for change period.

Distributions	Characteristics	MA	EWMA	TEWMA	MMTE	MTEM
Small to moderate shifts $\delta \subseteq [0.05, 1.00]$						
normal (0, 1)	EARL	151.6148	132.1552	122.5075	124.2654	**120.8449**
	EMRL	105.17	98.08	87.00	88.08	**85.00**
Laplace (0, 1)	EARL	209.3959	205.9504	156.7469	156.5361	**154.4167**
	EMRL	145.17	144.08	111.08	111.33	**108.00**
exponential (5)	EARL	151.5523	124.6213	113.4308	113.2830	**111.5881**
	EMRL	105.67	92.67	87.50	87.83	**86.00**
gamma (4, 1)	EARL	69.38646	67.03356	66.78054	68.34305	**65.64784**
	EMRL	48.20	47.00	48.00	49.80	**46.80**
Large shifts $\delta \subseteq [1.50, 3.00]$						
normal (0, 1)	EARL	**1.98334**	2.61300	5.15809	6.938162	5.07168
	EMRL	**1.50**	2.33	5.00	7.00	5.00
Laplace (0, 1)	EARL	**6.53437**	7.58859	7.10776	8.86130	6.93518
	EMRL	**5.00**	6.67	7.00	8.33	6.33
exponential (5)	EARL	**13.8073**	21.55739	28.12655	29.93638	27.73025
	EMRL	**10.33**	20.33	27.67	29.67	27.33
gamma (4, 1)	EARL	**1.19403**	1.64939	3.86169	5.54439	3.93161
	EMRL	**0.75**	1.25	3.50	5.50	3.50

Note: The lowest EARL and EMRL values are displayed in bold letters

5- Application

This section uses actual data to compare the success of various charts. In this study, two actual data sets are studied: CO_2 emission and the remission times associated with bladder cancer patients. These data are used to construct the proposed MA, EWMA, TEWMA, MMTE, and MTEM control charts. The statistical values are fewer than or equal to the UCL or larger than or equal to the LCL. The procedure is referred to be under control. If the calculated statistic exceed the UCL or are lower than the LCL, it is called an out-of-control process.

5-1- Carbon Dioxide (CO_2) Emissions

Monthly measurements of carbon dioxide (CO_2) emissions from natural gas sources in Thailand were taken from January 2019 through December 2023 [27]. These measurements were expressed in 100,000 tons. The 60 observations that make up the dataset have a mean of 65.10 and are distributed normally. The recorded values are as follows: 67.17, 65.43, 74.51, 72.47, 76.86, 71.28, 70.98, 67.60, 66.23, 71.05, 70.28, 63.53, 66.64, 63.89, 71.79, 60.58, 63.40, 62.77, 67.87, 62.43, 62.90, 61.85, 62.63, 60.00, 59.81, 59.81, 74.63, 65.93, 73.19, 66.26, 61.99, 61.36, 60.74, 61.82, 61.27, 57.86, 59.92, 59.39, 72.70, 62.74, 65.24, 65.14, 63.07, 61.54, 60.49, 55.25, 59.74, 55.56, 55.15, 56.72, 68.56, 70.95, 78.04, 71.60, 72.41, 70.28, 65.14, 64.02, 62.88, and 60.47.

An illustration of the findings from the control chart analysis can be found in Figure 6. The first chart to signal out of control is the MTEM chart, which detects a process change at 4th observation. After the MMTE and EWMA charts are provided at 5th observation, the TEWMA chart follows at 6th, and the MA chart occurs at 7th. These findings demonstrate that the MTEM chart outperforms the others in terms of detection of changes in CO_2 emissions.

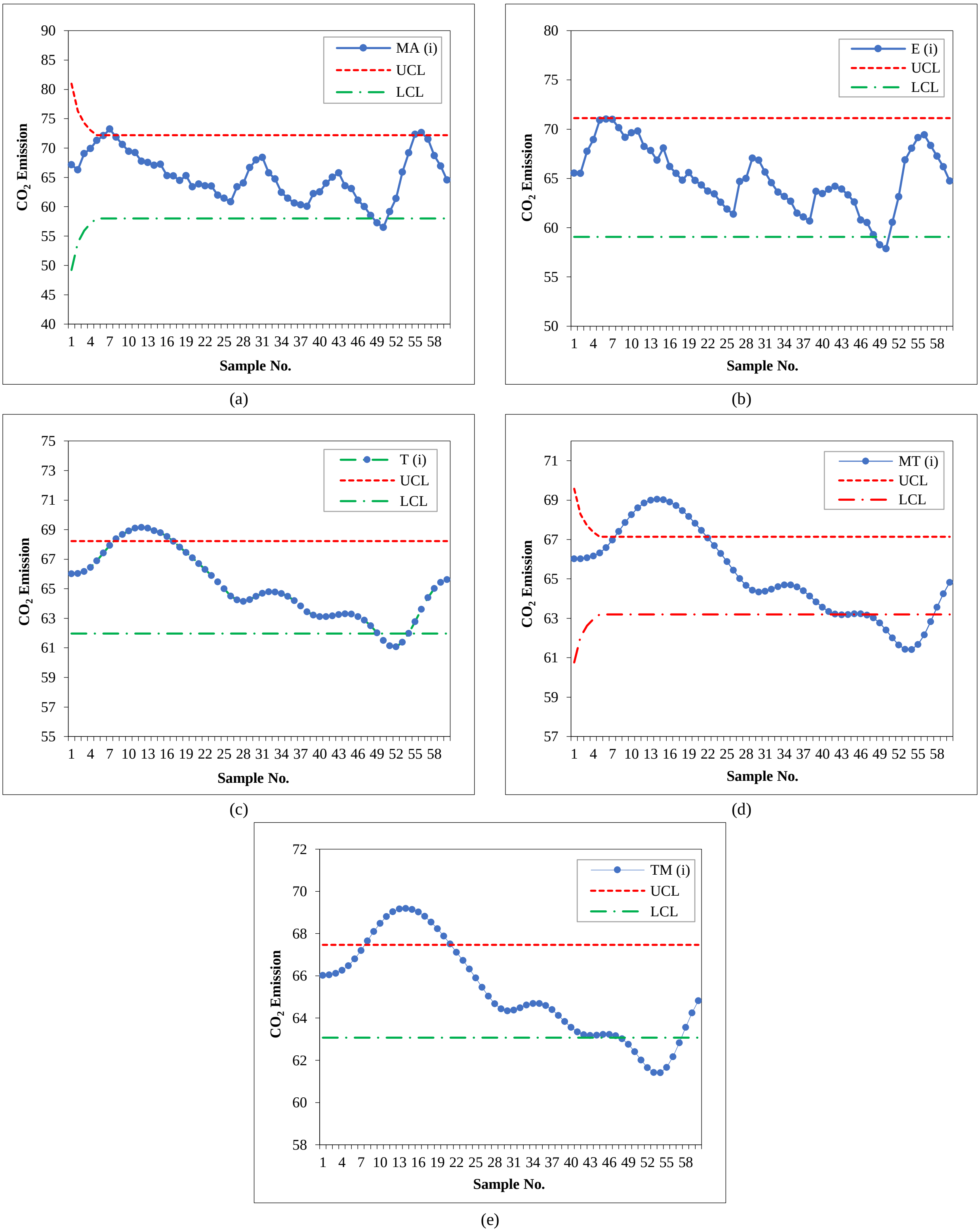


Figure 6. The plot of CO_2 Emissions in Thailand (2019–2023) data: (a) MA chart, (b) EWMA chart, (c) TEWMA chart, (d) mixed MMTE chart, and (e) proposed MTEM chart

5-2- Remission Times of Patients with Bladder Cancer

This dataset shows how long bladder cancer patients have been in remission, measured in months [28]. The 128 observations that make up the dataset have a mean of 9.21 and are distributed exponentially. The recorded values are as follows: 0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 6.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.61, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, and 22.69.

Figure 7 displays the investigation outcomes of remission times data. The MTEM chart signals a change at 4th observation, followed by the MMTE chart at 5th. In contrast, the EWMA chart identified the change much later, at 85th observation, while the MA and TEWMA charts failed to signal any abnormalities. These outcomes highlight the superior efficacy of the MTEM chart in modification detection, surpassing the MA, EWMA, TEWMA, and MMTE charts.

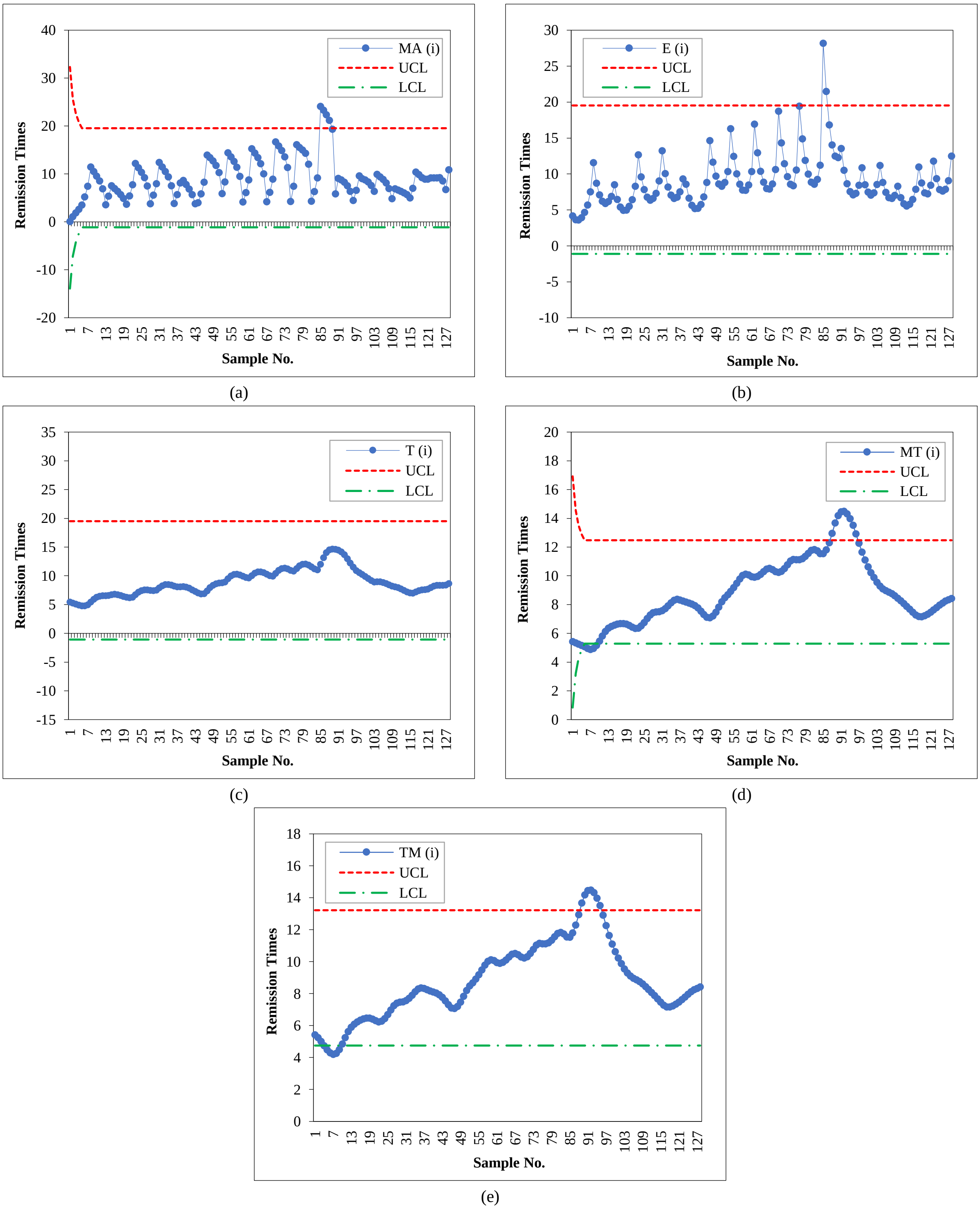


Figure 7. The plot of remission times data: (a) MA chart, (b) EWMA chart, (c) TEWMA chart, (d) mixed MMTE chart, and (e) proposed MTEM chart

The performance outcomes of the MTEM chart applied to two actual data sets demonstrate that it is far stronger at discovering changes in the procedure. However, the competing control charts on both sets of real data appear to have distinct capabilities. The difference may be due to change in the mean with growth in variance after the inflection point or to a property of the actual data distribution, such as skewness or autocorrelation.

6- Conclusion

This study proposed the MTEM chart for effective process monitoring, combining elements from both the TEWMA and MA charts. The MA, EWMA, TEWMA, and MMTE charts were measured against the proposed MTEM chart for both symmetric and non-symmetric distributions in term ARL and MRL. Its run-length characteristics were assessed through MC strategy. The ARL in control (ARL_0) was determined to be approximately 370. In addition, the EARL and EMRL values were also analyzed across a range of shift magnitudes. These results point out that the proposed MTEM chart was highly sensitive to process shifts of small to moderate magnitude across both symmetric and non-symmetric distributions, as reflected by the lowest ARL_1 and MRL values. For larger changes, the MA chart performed the best compared to the other charts. The TEWMA-MA chart exhibits reduced efficiency in detecting large shifts, as it begins with a moving average (MA) and subsequently applies three stages of exponential weighting. This sequential weighting process causes the statistic to adapt slowly. In the presence of a large change, the statistic responds slowly. Further, the MMTE chart is conceptually similar to the MTEM chart but differs in structure. Its design involves smoothing followed by re-averaging, which can lead to a significant loss of information. Consequently, the MMTE chart demonstrated a slightly slower performance compared to the presented chart.

This study also examined the relationship among the parameters w, ARL_1, and control limit constant of the MTEM chart (L_5), revealing that an increase in parameter w leads to higher L_5 values and lower ARL_1 values. Furthermore, the examination of the effects of varying the λ parameter on ARL_1 and L_5 revealed that an increase in λ parameter leads to higher values of L_5. Furthermore, ARL_1 gradually declines as λ and the process shift's magnitude increase. In order to assess the effectiveness of the charts, we used actual data on Thailand's carbon dioxide emissions and the remission periods within bladder cancer. Application to real data confirms that the MTEM technique has the highest sensitivity and efficiency in responding to process abnormalities among the MA, EWMA, TEWMA and MMTE control charts. The simulation results and empirical applications demonstrate the effectiveness of the proposed method for slight to modest shifts under symmetric and asymmetric distributions.

Simultaneously, the authors accomplished a comparison the MTEM proposed chart with EWMA-MA [16] and MEWMA-MA [21] for normal (0, 1) distribution, which had the following parameters: $ARL_0 = 370$, $w = 5$, and $\lambda = 0.25$. The outcomes demonstrated that the MTEM proposed control was superior to the EWMA-MA, and MEWMA-MA charts for parameter change ($-0.50 < \delta < 0.50$). Conversely, the MEWMA-MA outperformed the other chart for parameter change ($\delta \leq -0.75$ and $\delta \geq 0.75$). The results of the comparison between the MTEM and EWMA-MA charts suggested that MTEM outperformed EWMA-MA in the range of variation from -0.75 to 0.75. Finally, this proposed chart recommends selecting effective quality control charts in various scenarios. The point of this investigation can be broadened in future research to include asymmetric or alternative distributions. Additionally, Actual data from various sectors, including manufacturing, finance, transportation, and medicine, can be used to verify the MTEM chart's practicality

7- Declarations

7-1- Author Contributions

Conceptualization, S.S.; methodology, P.S.; validation, Y.A.; formal analysis, P.S.; investigation, P.S.; resources, Y.A.; writing—original draft preparation, P.S. and Y.A.; writing—review and editing, S.S.; visualization, P.S.; supervision, S.S.; project administration, Y.A. and S.S.; funding acquisition, S.S. All authors have read and agreed to the published version of the manuscript.

7-2- Data Availability Statement

The data presented in this study are openly available in Energy Policy and Planning Office [27] collected the CO_2 Emissions in Thailand dataset, and Shanker [28] collected the dataset of remission times in patients with bladder cancer.

7-3- Funding and Acknowledgements

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7-4- Institutional Review Board Statement

Not applicable.

7-5- Informed Consent Statement

Not applicable.

7-6- Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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