Available online at www.ijournalse.org

Emerging Science Journal

(ISSN: 2610-9182)

Vol. 10, No. 3, June, 2026

An Empirical Analysis of the Relationship Between the Omega Ratio and Yield Skewness in European Government Bonds

Attila Bányai [1], Tibor Tatay [2*], Gergő Thalmeiner [3], László Pataki [4, 5]

[1] *Doctoral School of Economic and Regional Sciences, Hungarian University of Agriculture and Life Sciences, Gödöllő H-2100, Hungary.*

[2] *Department of Statistics, Finances and Controlling, Széchenyi István University, Győr 9026, Hungary.*

[3] *Department of Investment, Finance and Accounting, Hungarian University of Agriculture and Life Sciences, Gödöllő H-2100, Hungary.*

[4] *Doctoral School of Management and Business Administration, John von Neumann University, Budapest H-1117, Hungary.*

[5] *Faculty of Social Sciences, Eötvös Lóránd University, Budapest H-1117, Hungary.*

Abstract

This study examines the empirical relationship between the Omega ratio and yield skewness in European sovereign bond markets, addressing whether distributional asymmetry is systematically reflected in Omega-based performance evaluation. The analysis is guided by two research questions: whether a statistically significant association exists between the Omega ratio and yield skewness across different time horizons, and whether this relationship exhibits cross-country heterogeneity consistent with a core–periphery structure. Using daily data for 10-year government bonds from 27 European countries over the period 2015–2025, we construct constant-maturity total returns and apply a robust Omega ratio formulation with inflation-adjusted thresholds. Yield skewness is measured using time-adjusted daily yield changes. The empirical strategy combines rolling-window correlation analysis, hierarchical clustering based on Kendall's τ, and Independent Component Analysis to capture both short-term dynamics and latent structural patterns. The results provide strong and consistent evidence of a significant relationship between the Omega ratio and yield skewness across short-, medium-, and long-term horizons, confirming that the Omega ratio captures meaningful aspects of return asymmetry in fixed-income markets. Importantly, the findings reveal pronounced regional heterogeneity: core and Northern European markets exhibit stable positive associations, while several peripheral and emerging markets display weaker or negative relationships. These results imply that Omega-based performance measures reflect not only statistical asymmetry but also underlying differences in market liquidity, risk premia, and institutional structure. Overall, the study highlights the relevance of distribution-sensitive performance measures for sovereign bond evaluation and contributes novel evidence from the European fixed-income context.

Keywords:

Omega Ratio;
Yield Skewness;
European Sovereign Bonds;
Risk-Adjusted Performance.

Article History:

Received:	02	December	2025
Revised:	28	April	2026
Accepted:	04	May	2026
Published:	01	June	2026

1- Introduction

In financial markets, return distributions are rarely normal, and they often display pronounced skewness and excess kurtosis. Skewness is particularly informative: positive skewness implies infrequent but potentially large gains, whereas negative skewness reflects exposure to rare but severe losses. Accordingly, the sign of the Omega–skewness relationship provides direct information on whether Omega captures desirable or undesirable forms of asymmetry. Empirical evidence from U.S. stock indices, for example, shows a strong rejection of the normality assumption and highlights the importance of higher-order moments in return distributions [1-3]. This suggests that performance measures relying only

* **CONTACT**: tatay@sze.hu

DOI: https://doi.org/10.28991/ESJ-2026-010-03-031

on the first two moments of returns, such as the Sharpe ratio, or related downside-focused metrics like the Sortino or Calmar ratios, may fail to capture important asymmetric behavior [4]. These limitations motivate the use of alternative measures such as the Omega ratio, which is explicitly based on the entire return distribution rather than only mean and variance. This raises the question of whether Omega systematically reflects the skewness of returns.

A growing number of studies have investigated this relationship. Metel et al. (2017) [5] showed that when return distributions are asymmetric, Omega and Sharpe ratios no longer rank portfolios equivalently, with Omega favoring right-skewed portfolios. Bernard et al. (2023) [6], in a comprehensive meta-analysis, confirmed these findings and reported that higher Omega values are generally associated with higher skewness, in line with earlier results by Metel et al. (2017) [5], and van Dyk et al. (2014) [7] further demonstrate that Omega can enhance performance evaluation relative to traditional measures because it is sensitive to distributional features such as skewness and kurtosis and can track their variation over time.

Nevertheless, the magnitude and stability of the omega–skewness relationship remain debated. Kazemi et al. (2004) [8] found that higher skewness and kurtosis exert relatively little influence on Omega unless the threshold is set well below the mean return. This suggests that the strength of the Omega–skewness relationship may be contingent on methodological choices such as the threshold level. Bi et al. (2019) [9] further contributed to this debate by introducing higher-order Omega measures explicitly tied to third and fourth moments, reinforcing that asymmetry remains central to Omega's interpretation. Optimization is also increasingly seen as an area where the Omega ratio offers advantages over traditional mean-variance or Sharpe-ratio-based models. By capturing the entire return distribution, Omega provides a more comprehensive framework for complex and non-normal assets [10], and extended formulations have been shown to combine this with mean–risk considerations to achieve improved risk properties [11]. It is also crucial to highlight that for symmetric return distributions such as normal or elliptic cases, the Omega ratio does not provide additional information beyond the Sharpe ratio, as both lead to the same optimal portfolio [12].

Overall, this body of research suggests that while Omega is sensitive to skewness, the degree and consistency of this relationship remain a subject of debate. While evidence points toward a positive correlation, especially in right-skewed contexts, the effect may weaken under certain parameterizations. Despite these advances, direct evidence on how Omega relates to asymmetry in sovereign fixed-income markets remains scarce. Prior studies predominantly examine equities, indices, hedge funds, commodities, or other asset classes [13] and typically do not test whether the Omega–skewness relationship is stable across countries or across investment horizons. European government bonds provide a particularly informative setting because yield dynamics embed policy shocks, liquidity conditions, and sovereign risk repricing that can generate persistent asymmetry. This study addresses this gap by providing the first systematic cross-country evidence on the relationship between the Omega ratio and yield skewness in European sovereign bond markets, using daily 10-year yields for 27 countries and rolling horizons (50, 100, and 200 days). Accordingly, we formulate two hypotheses to guide the analysis:

- ***H1:*** The Omega ratio and yield skewness are associated across short-, medium-, and long-term horizons in European sovereign bond markets.
- ***H2:*** A positive association is expected between the Omega ratio and yield skewness across European sovereign bond markets, though the strength may vary across different markets.

The remainder of the paper is organized as follows. Section 2 reviews the European sovereign bond market context and highlights key sources of cross-country heterogeneity. Section 3 presents the data and methodology, including the construction of constant-maturity total returns, the Omega and skewness measures, and the dependence and clustering techniques. Section 4 reports the empirical results. The final section concludes with implications and directions for future research.

2- Government Bond Markets and Their Heterogeneity in the European Region

The bond market is one of the most important pillars of the modern financial system. Its significance lies in the fact that it directly connects creditors and debtors and provides an efficient channel for the flow of funds. Investors can place their capital in safe, low-risk instruments, while debtors can obtain funds from a transparent and liquid market [14]. Government bonds carry particular weight in the financial system as a whole. These instruments are the main tools for managing public debt, but they are also one of the safest elements of investor portfolios. According to research by Fernández-Gallardo & Payá (2025) [15], excessive debt burdens can have serious consequences, such as deepening recessions, curbing investment, posing deflationary risks, and reducing the supply of credit.

The bond market serves dual roles in fiscal and financial spheres, playing a crucial role in coordinating monetary policy [16]. The results of Petlele & Buthelezi's (2024) [17] studies emphasize the importance of considering bond maturities and yield spreads when assessing their economic impact. Policymakers are advised to maintain stable and predictable monetary and fiscal policies to minimize uncertainty in interest rate movements and borrowing costs. Randl et al. (2025) [18] examined the pricing of currency-hedged government bond portfolios. They documented significant diversification benefits for investors due to imperfect correlation between term structure movements across bond markets, even when focusing only on developed markets and simple strategies such as GDP-weighted or equal-weighted portfolios. Moodley et al. (2024) [19] examined the impact of market-level investor sentiment on the returns of government bond indices of different maturities under varying market conditions. Their results suggest that the government bond market is adaptive and exhibits alternating efficiency. Nneka et al. (2025) [20] examined the impact of bond market development on economic growth in developing countries. The results show that government bond capitalization, trade openness, and inflation have a positive effect on economic growth, while corporate bond capitalization and domestic credit to the private sector have a negative effect on economic growth.

Stoupos & Kiohos (2022) [13] emphasized that one of the most important elements of European financial integration is the gradual convergence of bond markets. Their findings also highlight that the integration of Eastern European member states is much more heterogeneous. They showed that the core markets of the euro area play a decisive role, while peripheral economies are more sensitive to international financial shocks [21]. Balli (2009) [22] used multivariate GARCH models to examine the evolution of integration over time. The research showed that although European bond markets are closely linked, individual countries respond differently to global financial events. Barrios et al. (2019) [23] also confirmed that international factors, in particular global risk assessment, play a key role in explaining yield differentials. Eijffinger & Pieterse-Bloem (2023) [24] used a multidimensional factor model to approximate the drivers of euro area bond spreads. Gabrisch & Orlowski (2010) [25] showed that government bonds in Central and Eastern European countries converged significantly with euro-area yields after accession. Bernoth et al. (2012) [26] pointed out that government bond yields include a risk premium. Higher levels of government debt can therefore lead to higher yields, which can act as a disciplinary force on fiscal policy.

Market sensitivity to external shocks has been confirmed by several studies. Abad & Chulia (2013) [27] showed that U.S. and European monetary policy surprises have different effects on government bond markets. According to their research, market integration is a more important factor than the nature of the shock itself. Warin & Stojkov (2023) [28] used entropy methodology to examine the disciplinary role of European bond markets, while Afonso (2010) [29] pointed out that growth prospects and fiscal positions are closely related to bond yield developments.

The government bond market is shaped simultaneously by macroeconomic processes, fiscal decisions, monetary measures, and changes in the global risk environment. The examination of government securities is not only justified from a scientific point of view. It has significant implications for both investment and policy, as these instruments form the basis of government financing and are also prominent as safe components of investor portfolios. The varying changes in bond yields observed in European countries provide an opportunity to analyze the optimization of government bond portfolios. The literature provides sufficient evidence of the existence of heterogeneity. What makes our study novel is that we evaluate investment opportunities in European government bonds using the Omega ratio.

3- Materials and Methods

To ensure transparency and reproducibility, this section follows a structured, step-by-step methodological framework, summarized in Figure 1. The figure provides an overview of the complete analytical pipeline, from raw yield data to the final interpretation of the Omega–skewness relationship across European sovereign bond markets.

When selecting European government bonds, we exclusively focused on the 10-year maturity for two primary reasons. First, the 10-year yield is widely regarded as the key benchmark in the European bond market. Second, it provides a balance between short-term liquidity and long-term stability, making it a reliable indicator of investor sentiment and economic expectations. During our selection process, we included almost all EU member states, along with several major non-EU issuers, in order to ensure coverage of the vast majority of the European government bond market. Liquidity was another important criterion; therefore, a few countries were excluded from the sample. The analyzed period spans from August 28, 2015, to August 28, 2025, covering ten years and 27 countries in total, of which four are non-EU states. Non-EU issuers (Iceland, Norway, Switzerland, and the United Kingdom) are included to broaden the coverage of the European sovereign bond universe, while Cyprus, Estonia, Latvia, and Luxembourg are excluded due to limited data availability or low market liquidity. The daily yields were obtained from Investing.com's historical database (see Table 1).

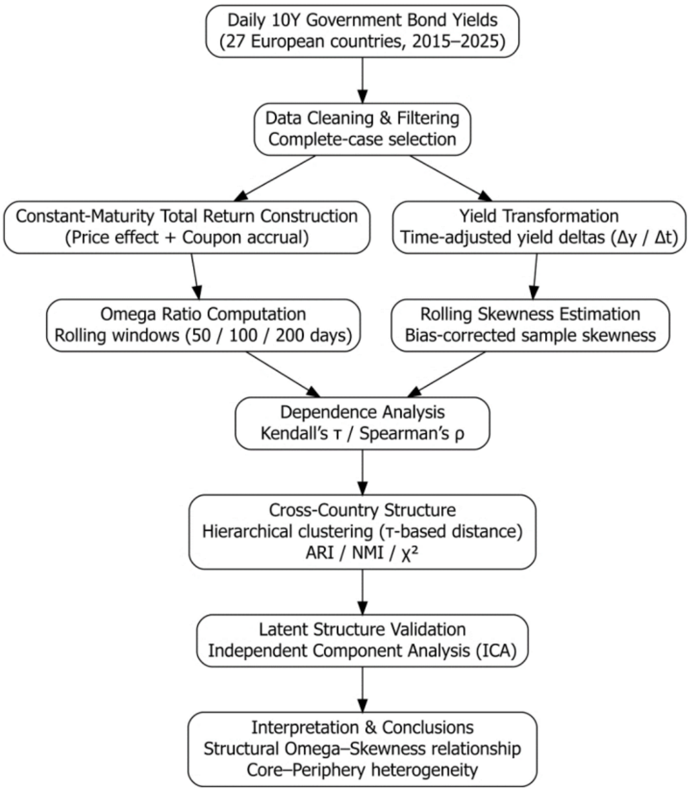

Figure 1. **Summarizes the methodological workflow**

Table 1. **Overview of selected countries with 10-year government bonds**

Country	EU member	Euro area
Austria	x	x
Belgium	x	x
Bulgaria	x	
Croatia	x	x
Czech Republic	x	
Denmark	x	
Finland	x	x
France	x	x
Germany	x	x
Greece	x	x
Hungary	x	
Iceland		
Ireland	x	x
Italy	x	x
Lithuania	x	x
Malta	x	x
Netherlands	x	x
Norway		
Poland	x	
Portugal	x	x
Romania	x	
Slovakia	x	x
Slovenia	x	x
Spain	x	x
Sweden	x	
Switzerland		
United Kingdom	until January 31, 2020	

The initial dataset of 74,766 daily pairwise observations contained a significant number of incomplete records, a common challenge in cross-country financial data. A large volume of missing data can substantially compromise statistical integrity; therefore, we carefully evaluated several imputation methods. After considering approaches such as Last Observation Carried Forward (LOCF), daily cross-sectional means, and K-Nearest Neighbors (KNN), we concluded that these methods introduced an unacceptable level of data distortion and artificial volatility, as shown in Table 2. To ensure the highest degree of data integrity, we adopted the complete-case analysis method, relying exclusively on observations where data for all variables was available. Although this reduced the dataset to 45,630 observations, we prioritized the authenticity of our data. By avoiding interpolation or the carryover of previous-day data, our analysis remains a true reflection of the daily market dynamics and avoids the creation of synthetic relationships between yields.

Table 2. Data Preprocessing and Missing-Data Diagnostics

Method	Observations	Std. Dev. (× original)	Skewness	Kurtosis
Original (daily, pairwise)	74,766	1.0000	1.4826	555.8341
Complete-case (daily)	47,292	1.2267	0.7511	256.3669
LOCF-unbounded (daily)	96,096	0.8821	1.6875	714.3984
Daily Cross-sectional Mean	96,096	21.3307	−0.0093	76.0719
Daily KNN (k=5)	96,096	13.9705	−0.0515	89.5699
Daily Regression (MICE)	94,920	11.1851	−0.7073	119.8667
Weekly (last observation)	14,580	2.3384	0.7997	144.0959

Unlike perpetual securities such as equities, bonds cannot be directly analyzed using simple or log returns of yields expressed as market prices. Fixed-income instruments have a finite maturity, and their valuation depends not only on market yield levels but also on coupon payments and the time structure of cash flows. A change in yield therefore affects both the discounting of future cash flows (price effect) and the accrual of coupon income (income effect). To obtain a financially consistent daily return measure from yield data, it is necessary to construct a synthetic bond framework that incorporates these features rather than relying on standard return formulas designed for securities without maturity.

We approximate bond total returns from a yield time series using a rolling constant-maturity par-bond framework with day-count accrual. On each date t, we assume a 10-year bond issued at par whose coupon equals the previous period's yield (y_{t-1}). At date t, we reprice this bond using the current yield (y_t), while the maturity is reset to 10 years (constant-maturity assumption).

Let the coupon be $C = y_{t-1}$, maturity $T = 10$, and face value normalized to $F = 1$;

The bond price is:

$$P(y_t) = \begin{cases} \frac{C\left(1-(1+y_t)^{-T}\right)}{y_t} + (1+y_t)^{-T}, & y_t \neq 0 \\ CT + 1, & y_t = 0 \end{cases} \quad (1)$$

where, $\mathrm{C} = \mathrm{y}_{t-1}$ is the coupon rate set at the previous observation, $\mathrm{T} = 10$ is the maturity in years, $\mathrm{y_t}$ = the yield-to-maturity at time t.

The period's total return is

$$\mathrm{R_t} = (\mathrm{P(y_t)} - 1) + \mathrm{C\Delta t} \quad (2)$$

where Δt is the year fraction between dates and yields y_t are simple annual rates.

Iterating this procedure over the yield series yields a synthetic constant-maturity total-return series that captures both price effects from yield changes and income effects from coupon accrual, thereby enabling the conversion of daily yields into daily total returns for Omega ratio evaluation. This continuous repricing produces a smoother synthetic time series, as the constant-maturity assumption avoids the discontinuities that would arise from rolling over individual bonds with different maturities [30]. While this approach does not increase the number of available observations, it reduces structural breaks and ensures that missing data points do not create artificial jumps in the return series.

The Omega ratio was originally introduced by Keating & Shadwick (2002) [31] as a cumulative distribution function (CDF) based performance measure that captures the entire return distribution instead of relying solely on mean–variance trade-offs. This formulation represented a significant innovation, as it explicitly incorporates distributional asymmetry and tail behavior. In addition, subsequent research has pointed out that the original Omega can be sensitive to sampling noise and may lack robustness under different market conditions [32]. To address these challenges, Kapsos et al. (2014) [33] proposed a robust optimization–based version of the Omega functional. While not identical to the original definition, their approach is consistent with its distributional perspective and is designed to enhance stability and tractability in portfolio selection. In our study, we adopt the formulation of Kapsos et al. (2014) [33] as the baseline, while also reporting results based on the standard threshold-based Omega for transparency and comparability. The robust formulation can be expressed as:

$$\Omega(\mathrm{r},\tau) = \frac{\mathrm{E(r)}-\tau}{\mathrm{E}^{+}(\tau-\mathrm{r})} + 1 \quad (3)$$

where, r denotes the portfolio return random variable, τ is the chosen threshold (minimum acceptable return), $E(r)$ is the expected return, and $E^{+}(\tau - r)$denotes the expected shortfall below the threshold, taken only over the loss region $(r < \tau)$.

Our correlation analysis employs a rolling-window approach, whereby each Omega ratio is calculated using moving data windows of 50, 100, and 200 days. This method allows us to examine short-, medium-, and long-term market dynamics. A rolling analysis of time series is particularly useful in this context, as it relaxes the assumption that parameters remain constant over the entire sample and instead captures potential structural changes over time [34]. The numerator of the Omega ratio is defined as the difference between the moving average of the constant-maturity total-return series and the moving average of the daily euro area inflation rate, which measures real returns relative to inflation. The denominator corresponds to the moving average of shortfalls, defined as constant-maturity daily returns that fall below the daily inflation rate, thereby capturing downside risk. This methodology generates a dynamic time series, providing detailed insight into how the risk–return profile evolves over time.

As the source data for euro area inflation, we employed the ECB's HICP – Overall Index database, which reports the euro area's year-on-year (YoY) rate on a monthly basis, as provided by Eurostat. Since these are annual inflation rates, while our analysis focuses on daily transactions, we converted them into daily observations. To achieve this, we accounted for the exact number of calendar days between two consecutive dates, thereby ensuring that the daily inflation impact corresponds precisely to the actual time elapsed. This dailyized inflation rate also serves as the threshold parameter (τ) in the Omega ratio, which is crucial for consistency: the literature emphasizes that the choice of threshold strongly affects the interpretation of Omega [35], and using inflation as a benchmark ensures both economic relevance and comparability.

$$\pi_{t_1 \to t_2} = (1 + \pi_{\text{annual}})^{d/365} - 1 \quad (4)$$

where, $\pi_{\mathrm{t}_1 \to \mathrm{t}_2}$ is compounded inflation over the period $\mathrm{t}_1 \to \mathrm{t}_2$, π_{annual} is annual inflation rate, and d is the number of days between the two dates.

While the previously introduced constant-maturity total-return series is appropriate for calculating measures such as the Omega ratio, it is not suitable for the skewness analysis, since it is an interpolated and smoothed series rather than a directly observed market yield. As a result, it fails to capture the discrete, high-frequency yield movements that are essential for an unbiased estimation of skewness. To address this issue, we introduce the time-adjusted yield delta, which is the second key element of our correlation analysis and forms the basis of the rolling-window skewness calculation. It is computed as the change in the 10-year sovereign yield divided by the exact number of calendar days between observations, thereby producing a dailyized yield change that is directly comparable across periods of varying length and robust to irregular observation gaps such as weekends, holidays, or the reduced number of observations resulting from the complete-case analysis. This normalization ensures consistency in the estimation of higher-order moments such as skewness. Without this normalization, using simple yield subtractions on data with irregular gaps (such as holidays or missing trading days) creates non-uniform changes. A change over three days will naturally be much larger than a one-day change. These larger, infrequent values bias the distribution of the data, leading to an inaccurate skewness estimate. Normalizing the change by the number of days mitigates this by making every data point a comparable daily change.

$$\Delta y_t^{\text{daily}} = \frac{y_t - y_{t-1}}{\Delta t} \quad (5)$$

where, y_t is the yield observed on day t, y_{t-1} is the yield observed at the previous observation, and Δt is the number of calendar days elapsed between the two observations.

Once the daily yield deltas are computed, the rolling skewness is obtained using the bias-corrected sample formula

$$\widehat{\text{Skew}} = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s}\right)^3 \quad (6)$$

where, n denotes the number of observations, $\bar{x}$ is the sample mean, and s is the sample standard deviation (calculated using denominator $n - 1$).

To address the central objective of this study, we examine the relationship between the calculated Omega ratios and yield skewness values across different countries and rolling windows (50, 100, and 200 days). Given the distributional properties of financial returns, we hypothesized that the data would likely violate the assumptions of Pearson correlation, namely linearity and normality, due to the presence of nonlinear dependencies and outliers. Therefore, we systematically evaluated which correlation measure provides the most reliable characterization of these relationships. In this context, both Spearman's rank correlation and Kendall's τ were applied and compared, allowing us to assess which approach better captures the dependence structures most relevant to our problem.

To examine the structural similarities and differences between assets, we apply a hierarchical agglomerative clustering framework. In contrast to distance measures based on Euclidean metrics, we employ a dissimilarity measure derived from Kendall's τ correlation coefficient, defined as $D = 1 - \tau$. This approach captures rank-based dependence structures that are robust to outliers and non-normal return distributions, making it particularly suitable for financial time series [36]. The resulting dissimilarity matrix is used as input for Ward's minimum variance method (Ward.D2 linkage), which iteratively merges clusters in order to minimize the increase in within-cluster variance at each step. The procedure is implemented separately for the Omega ratio and yield skewness across each rolling window. To determine the appropriate number of clusters, we rely on the average silhouette criterion, which balances compactness and separation by maximizing the mean silhouette width. The resulting partitions are then compared across the two perspectives (Omega vs. yield skewness) using adjusted Rand index (ARI) and normalized mutual information (NMI), complemented by chi-squared tests of the contingency tables to assess the statistical significance of the observed cluster associations.

We consider it important to complement the correlation and clustering results with an additional dimension-reduction technique. While the correlation analysis and hierarchical clustering identified static cross-country patterns in the Omega–skewness relationship, these approaches do not explicitly reveal the latent factors that may jointly drive such structures. Independent Component Analysis (ICA) offers a suitable extension, as it decomposes the multivariate τ-vectors (50-, 100-, and 200-day horizons) into statistically independent components. By doing so, ICA highlights hidden axes of variation that may correspond to economically meaningful divides, most notably the distinction between core and peripheral markets. This method therefore allows us to verify whether the short-term vs. medium-term correlation regimes, already suggested by Kendall's τ and the clustering diagnostics, can be represented by a small number of independent sources, and whether these in turn reproduce the regional bifurcation observed elsewhere in the analysis. We use ICA instead of Principal Component Analysis (PCA) because it extracts statistically independent, rather than merely uncorrelated, factors, is well suited to non-Gaussian financial data, and better isolates latent drivers behind the Omega–skewness structure that covariance-based methods may obscure [37].

4- Results

4-1- Correlation Analysis

Diagnostic results consistently indicate that Kendall's τ is the most appropriate choice under stricter diagnostic thresholds, while Spearman's ρ gains relevance when a higher tolerance for influential observations is allowed. Normality tests including Shapiro–Wilk and the observed skewness patterns clearly invalidate Pearson's parametric assumptions, and high Cook's distance values confirm the presence of influential outliers that challenge the analysis. In addition, ties and the sizeable gaps between Pearson and Spearman coefficients point to predominantly non-linear but monotonic associations. Under the strictest cutoff (e.g., Cook's distance = 1 or 3), Kendall's τ was preferred in over 97% of cases, underscoring its superior robustness to ties and non-linearities. When the threshold was relaxed (e.g., Cook's distance = 5), however, Spearman's ρ became dominant, being selected in around 60% of all decisions, indicating that it may be the superior option when influential observations are retained.

Across all three rolling horizons (50, 100, and 200 days), Kendall's τ indicates a consistent and statistically significant association between the Omega ratio and yield skewness in most European sovereign bond markets, with clear regional differentiation (see Figure 2 and Table 3). Core and Northern European economies exhibit a uniformly positive and remarkably stable pattern: countries such as Germany, Norway, Austria, Ireland, and the Netherlands show persistent positive τ values across all windows, with the association typically strengthening at longer horizons. This collective behavior suggests that in more mature and liquid sovereign bond markets, the Omega ratio and yield skewness move in a stable and reinforcing manner, highlighting the structural nature of this relationship in core and Northern Europe.

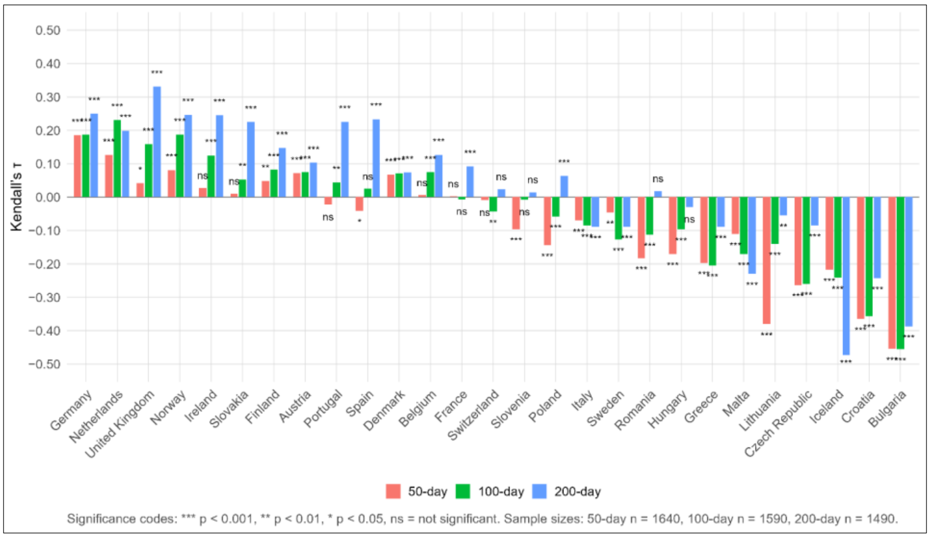

Figure 2. Kendall's τ Correlations Between Omega Ratio and Yield Skewness Across Countries

Table 3. Clustering quality and stability metrics across different numbers of clusters (k)

Window	K (clusters)	Mean Silhouette	Mean Cluster Size Entropy	ARI	NMI
50	2	0.2391	0.6221	0.8503	0.7720
50	3	0.2489	0.8131	0.7949	0.6953
50	4	0.1955	1.1222	0.4617	0.5296
50	5	0.1509	1.3760	0.2688	0.4944
100	2	0.2551	0.6365	0.4727	0.3572
100	3	0.2704	0.8655	0.4865	0.3276
100	4	0.2726	0.9742	0.5079	0.3503
100	5	0.1467	1.3833	0.2450	0.3602
200	2	0.3082	0.4927	0.0927	0.0459
200	3	0.2859	0.8754	0.2740	0.2722
200	4	0.2565	1.1652	0.1895	0.2859
200	5	0.2592	1.2629	0.1895	0.3175

In contrast, several peripheral and emerging economies show consistently negative associations, often of substantial magnitude. The steepest declines appear in certain Central and Eastern European markets, with similarly persistent negative signs in several Southern European and smaller economies. While the exact values differ, the pattern is highly coherent: these markets systematically exhibit asymmetric yield behavior that moves inversely with Omega-based assessments of return quality. This contrast with core markets suggests that negative Omega–skewness relationships are not idiosyncratic but reflect deeper structural divergences in liquidity, market depth, and sovereign risk pricing across Europe.

A smaller group of countries exhibits weak or near-zero correlations, regardless of horizon. These include several highly liquid markets where effect sizes remain negligible even when statistically significant, as well as some smaller markets where the estimated coefficients fluctuate around zero. Because these nonsignificant cases are scattered across both regions and horizons, without forming a coherent cluster, their interpretation is likely idiosyncratic, driven by data limitations, local shocks, or horizon-specific noise rather than any systematic divergence from continental patterns.

Taken together, the rolling-window results point to a structurally meaningful relationship between the Omega ratio and yield skewness across Europe. Positive associations cluster in core and Northern economies, while negative associations are concentrated in several peripheral and emerging markets. Although the sign of these relationships is stable across horizons, their magnitude varies as the window length changes, and the influence of outliers can modulate the statistical strength in particular markets. Nevertheless, the broad geographic segmentation and the consistency of the patterns across time windows underscore that the Omega–skewness relationship captures an enduring feature of European sovereign bond market behavior.

From an economic perspective, these correlation patterns indicate that the Omega ratio captures fundamentally different forms of return asymmetry across European sovereign bond markets. In core and Northern European economies, positive and stable Kendall's τ values suggest that higher Omega ratios are associated with favorable skewness, reflecting well-functioning, liquid markets where upside yield movements are more effectively rewarded. In contrast, the negative associations observed in several peripheral and emerging markets imply that skewness often arises from downside-driven yield dynamics, such as abrupt repricing events or liquidity shocks, which are penalized by the Omega measure. This asymmetry explains why the sign of the Omega–skewness relationship differs systematically across regions and provides direct support for Hypothesis H1.

4-2- Hierarchical Clustering

Across all three rolling windows, the hierarchical clustering results reveal a stable and economically meaningful structure in how European sovereign bond markets group together when classified by the Omega ratio and yield skewness. Although the numerical indicators are summarized in Table 3, the broader pattern is evident: a three-cluster configuration consistently provides the most balanced outcome. It achieves strong internal cohesion, avoids the excessively unequal cluster sizes seen in simpler two-cluster solutions, and prevents the over-fragmentation that appears when four or five clusters are imposed. In practice, this means that the three-cluster structure captures the major divisions between core, intermediate, and peripheral markets while retaining enough granularity to reflect real differences in yield behavior and performance asymmetry.

The fact that this structure persists across short-, medium-, and long-horizon windows suggests that it is not an artefact of window length but reflects underlying, persistent features of European sovereign bond markets. The close correspondence between Omega-based and skewness-based partitions indicates that the two measures, although conceptually different, highlight similar cross-country patterns. This stability supports the interpretation that Omega and yield skewness encode related information about risk asymmetry, liquidity, and market structure. As a result, the three-cluster solution emerges not just as a statistical optimum, but as a robust representation of the regional and structural segmentation that characterises European fixed-income markets.

In the 50-day window, three clusters provide a well-defined structure without overfitting (Figure 3). Fewer clusters would compress economically distinct market groups into overly broad categories, while additional clusters would fragment the sample and create very small groups with limited interpretive value. With three clusters, the partition clearly separates core and northern European markets, peripheral and emerging markets, and a small intermediate group. The chi-squared test confirms that the Omega-based clusters and skewness-based clusters are strongly aligned at this horizon, and the contingency table shows that almost the entire cross-section falls along the diagonal. This reflects a tight short-term link between the two perspectives and suggests that daily yield asymmetry and Omega-based performance assessments follow a closely matching structure in the immediate horizon.

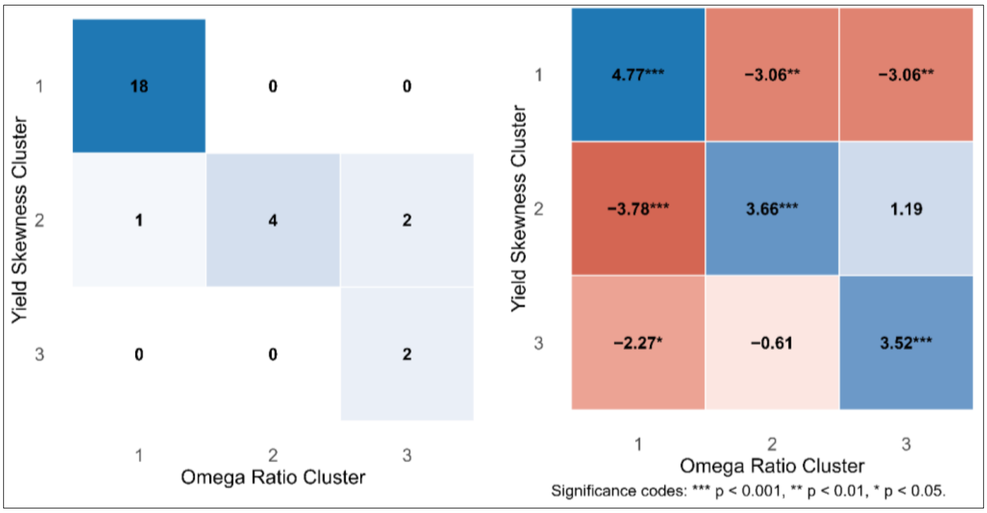

Figure 3. **Contingency heatmap and residual heatmap (standardized Pearson residuals) of Omega ratio clusters versus yield skewness clusters for the 50-day window**

At the 100-day horizon, the three-cluster structure again emerges as the most balanced solution, although the strength of the alignment weakens slightly (Figure 4). The main core cluster remains stable and continues to group together the large, liquid sovereign markets of Western and Northern Europe. Several countries, however, begin to drift toward alternative blocks in ways that are economically meaningful. Southern and Eastern European markets display more fluid cluster memberships, indicating that medium-term yield dynamics start to diverge from their short-term configurations. The chi-squared test still identifies a statistically significant relationship between the Omega and skewness partitions, but the diagonal mass is noticeably smaller than in the 50-day case, pointing to increasing heterogeneity as the time window lengthens.

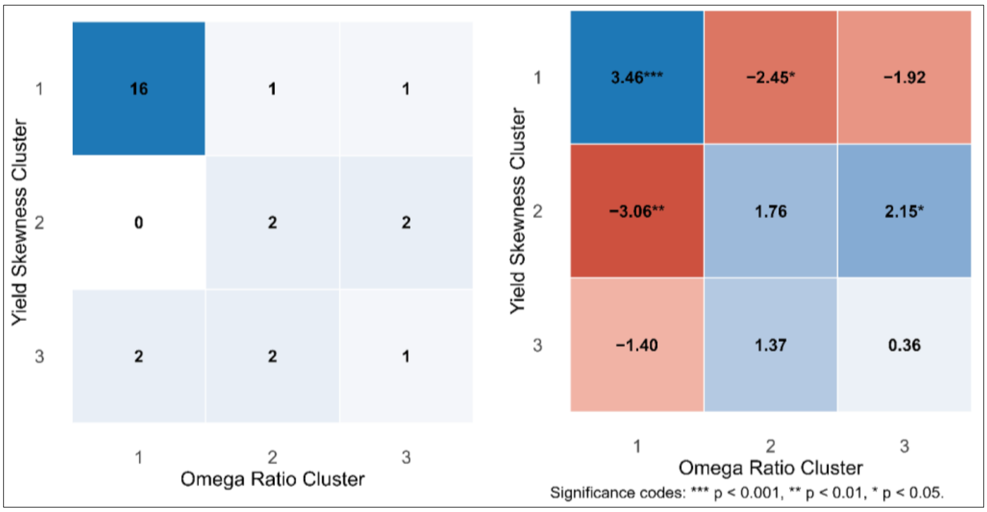

Figure 4. **Contingency heatmap and residual heatmap (standardized Pearson residuals) of Omega ratio clusters versus yield skewness clusters for the 100-day window**

By the 200-day window, the clustering structure becomes more diffuse while remaining interpretable (Figure 5). Three clusters continue to form the most coherent partition, but their composition is now more varied. The main core cluster persists, although some countries move in and out of this group depending on their longer-term dynamics. Several peripheral and emerging markets shift across clusters in ways that reflect liquidity differences, local risk premia, and country-specific repricing events. While the dependence between the Omega-based and skewness-based classifications remains statistically significant, the share of countries on the diagonal of the contingency table falls further. This indicates that long-horizon yield asymmetry and Omega performance begin to decouple in a subset of markets, particularly in those with more volatile or structurally heterogeneous sovereign risk characteristics.

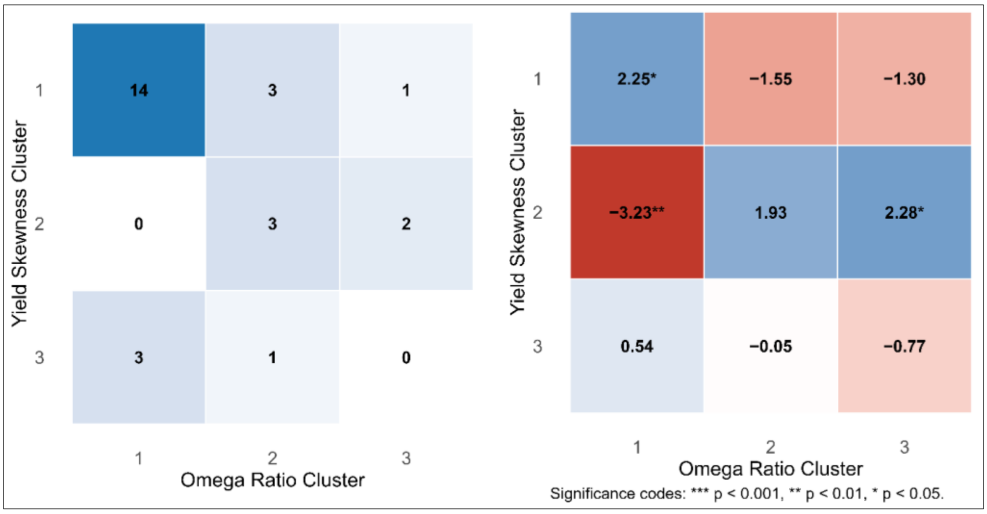

Figure 5. Contingency heatmap and residual heatmap (standardized Pearson residuals) of Omega ratio clusters versus yield skewness clusters for the 200-day window

The contingency heatmaps clearly illustrate this progression. At 50 days, the Omega and skewness clusterings align almost perfectly, producing a dominant central block of core markets and two sharply distinct peripheral groups. At 100 days, the association is still visible but more fragmented, with several economies gradually transitioning away from their short-term positions. By 200 days, the mapping becomes more complex, with multiple markets rotating across clusters and a growing presence of off-diagonal elements. These shifts are not random but follow recognizable regional lines: core euro area countries remain grounded in stable Omega–skewness regimes, while smaller and more vulnerable markets display more pronounced rotational behavior.

From a temporal perspective, the clustering results reveal a clear temporal pattern. The short-term structure is highly synchronised across Europe, the medium-term horizon introduces moderate divergence, and the long-term view brings out deeper regional and structural heterogeneity. This evolution underscores that the Omega–skewness relationship is strongest at short horizons and becomes increasingly shaped by country-specific factors as the window lengthens. The persistence of a large and stable core cluster, combined with the gradual dispersion of peripheral markets, highlights the importance of liquidity conditions, sovereign risk differentials, and institutional frameworks in shaping the long-term behavior of European government bond markets.

The clustering results further reinforce this interpretation by revealing that markets with similar Omega–skewness dynamics form economically coherent groups. The persistent separation between core, intermediate, and peripheral clusters suggests that the observed patterns are not driven by transitory noise but reflect structural differences in market depth, liquidity provision, and sovereign risk premia. From a portfolio perspective, this implies that European government bonds cannot be treated as a homogeneous asset class when performance is evaluated under distributional asymmetry. Instead, the Omega ratio highlights region-specific risk profiles that are relevant for diversification, risk budgeting, and regime-aware allocation strategies.

4-3- Independent Component Analysis (ICA)

To provide a complementary perspective on the second hypothesis (H2), we applied Independent Component Analysis (ICA) to the country-level Kendall's τ vectors across the 50-, 100-, and 200-day horizons (Table 4). Whereas correlation and hierarchical clustering revealed static co-movement and structural grouping, ICA allows us to uncover latent, statistically independent sources of variation. By imposing a two-cluster solution on the ICA scores, the method directly tests whether the extracted independent components capture the hypothesized core–periphery divide in European government bond markets.

Table 4. **Correlations between independent components and Kendall's τ measures (50-, 100-, 200-day horizons)**

Component	τ50	τ100	τ200
Independent Component 1	–0.7886 (p < 0.001)	–0.5660 (p = 0.0021)	–0.2178 (p = 0.2750)
Independent Component 2	–0.6068 (p = 0.0008)	–0.8071 (p < 0.0001)	–0.9729 (p < 0.0001)

The decomposition revealed two independent components with distinctly different temporal profiles. The first component is closely tied to short-term and transitional dynamics, as it aligns most strongly with the 50-day horizon and retains a meaningful, though weaker, link to the 100-day window. This pattern indicates that the first component primarily captures the immediate co-movement between Omega and skewness, which gradually fades as the horizon lengthens. In contrast, the second component is overwhelmingly dominated by long-horizon behavior. It shows its strongest association with the 200-day window and remains closely connected to the medium-term horizon as well, suggesting that it represents the persistent, longer-term structure of the Omega–skewness relationship.

The mixing matrix reinforces this interpretation: the first component loads most heavily on short-horizon dynamics, while the second component almost entirely reflects long-horizon variability (Table 5). The 100-day horizon displays notable loadings on both components, which positions it as a transitional regime connecting short-run and persistent effects. Taken together, these findings confirm that ICA successfully isolates two orthogonal sources of variation, corresponding to short-term adjustments and long-term structural co-movement in the Omega–skewness association.

Table 5. **ICA-Derived Linear Mixing Weights**

Horizon	Independent Component 1	Independent Component 2
τ50	–0.7739	–0.5954
τ100	–0.5554	–0.7920
τ200	–0.2138	–0.9547

When the ICA scores were subjected to k-means clustering with k = 2, the resulting partition broadly reproduced the geographic segmentation already suggested by Kendall's τ and the hierarchical clustering (Figure 6). Cluster 2 groups together the core and Northern economies, including Germany, Austria, Belgium, the Netherlands, Denmark, Finland, France, Sweden, and Switzerland, characterized by positive and stable τ values. Cluster 1, by contrast, is dominated by peripheral and Eastern European markets such as Bulgaria, Croatia, Greece, Hungary, Poland, Portugal, Romania, Slovakia, Slovenia, and Spain. This dichotomy reinforces the conclusion that the Omega–yield skewness association is structured along a core–periphery axis.

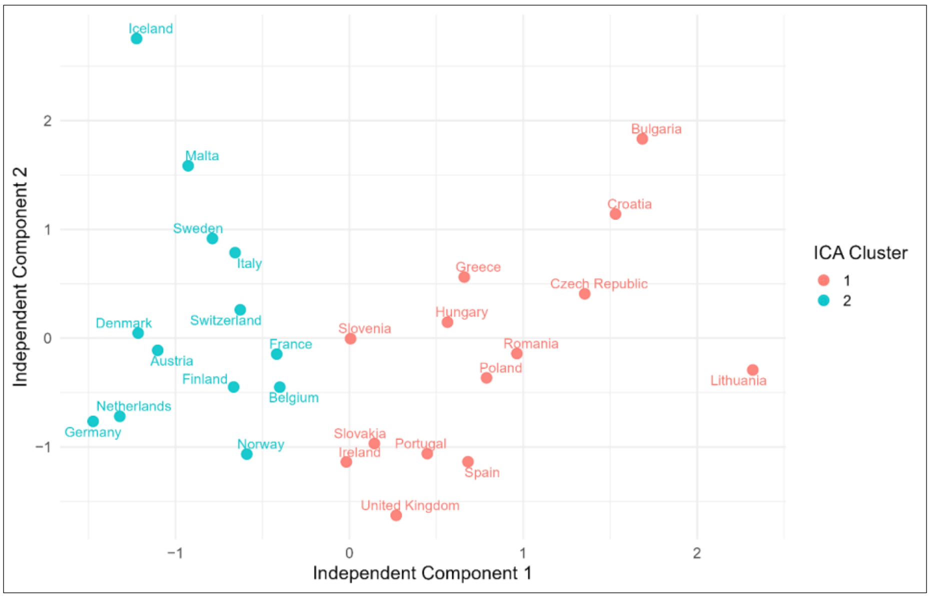

Figure 6. **Two-cluster ICA scores of Kendall's τ vectors (50–100–200 days), illustrating potential core–periphery patterns**

At the same time, the ICA results reveal important exceptions that enrich the interpretation. Ireland, despite its convergence in recent decades, aligns with the peripheral cluster, consistent with its intermediate position in the correlation analysis. The United Kingdom, typically considered a mature financial market, surprisingly loads with the periphery, likely reflecting its non-euro status and idiosyncratic post-Brexit risk dynamics. Conversely, Italy, Malta, and

Iceland are placed within the core cluster, despite earlier evidence of negative τ associations or higher volatility; this suggests that ICA captures latent factors that emphasize their long-horizon co-movement over short-term asymmetries.

Overall, the ICA findings provide partial empirical support for the second hypothesis (H2), showing that the Omega–skewness relationship is structured along a core–periphery axis. The two independent components extracted by ICA capture short- versus long-term dimensions of the relationship and reproduce the regional segmentation already suggested by the correlation and clustering results. Economically, this heterogeneity is likely linked to differences in liquidity provision, institutional setup, and cross-country risk premia across European markets. Meanwhile, the presence of systematic exceptions shows that regional classification is not purely mechanical but shaped by country-specific shocks. Thus, ICA not only corroborates the earlier analyses but also adds nuance by revealing the hidden factor structure underlying the observed asymmetries.

The ICA results provide additional insight into the temporal structure of the Omega–skewness relationship by separating short-term adjustments from longer-term structural co-movement. The dominance of the first independent component at shorter horizons suggests that transient market conditions and short-lived shocks play a key role in shaping near-term asymmetry. By contrast, the second component captures a persistent long-horizon factor that aligns closely with the core–periphery segmentation of European bond markets. This decomposition confirms that the observed heterogeneity is not merely cross-sectional but also inherently dynamic, offering partial support for Hypothesis H2 and highlighting the importance of horizon-specific interpretation.

5- Conclusions

This study provides compelling empirical evidence for a robust and statistically significant relationship between the Omega ratio and yield skewness in European government bond markets. Our findings consistently support the first hypothesis (H1), showing that this link is structurally embedded rather than a statistical artefact, given its persistence across short-, medium-, and long-term horizons. The association proved significant in over four-fifths of the country–horizon combinations, with the few nonsignificant cases scattered across markets and horizons, reinforcing that the Omega–skewness relationship captures an intrinsic feature of bond return distributions. Beyond its statistical strength, this evidence demonstrates that performance measures grounded in the full return distribution, rather than mean–variance trade-offs alone, are particularly relevant in fixed-income markets where asymmetries are structurally embedded in yield dynamics. From an investment perspective, the Omega ratio's sensitivity to distributional asymmetry provides a useful lens for distinguishing more stable, liquid core markets from more volatile peripheral markets, supporting risk budgeting, drawdown control, and allocation across regimes.

In contrast, the evidence partially supports the second hypothesis (H2) by demonstrating a clear cross-country heterogeneity that aligns with a core–periphery structure. Markets in Western and Northern Europe, characterized by deeper liquidity and greater institutional stability, show stable positive associations. Conversely, peripheral and emerging markets often exhibit negative or less consistent relationships, reflecting the influence of market microstructure and risk premium differences. To our knowledge, this is the first study to systematically document this distinct pattern in European sovereign bonds, thereby extending the literature on performance measures into the fixed-income domain. These findings carry important implications for both theory and practice: they highlight the need to incorporate distributional asymmetry into bond performance evaluation and underscore the importance for investors and policymakers of explicitly accounting for regional segmentation when assessing risk and allocating capital.

While a direct numerical comparison with earlier studies is not feasible due to fundamental methodological differences, the results of the present study can be meaningfully interpreted in light of the existing literature. Most prior empirical analyses of the Omega–skewness relationship focus on equity indices, commodities, or mixed-asset portfolios and typically rely on price-based returns and static or single-horizon performance measures [5-7]. In contrast, the present study examines sovereign bond markets using yield-implied total returns, rolling-window dynamics, and an inflation-adjusted threshold, which captures a different economic mechanism. Despite these differences, our findings are consistent with the core insight of the literature that the Omega ratio is sensitive to distributional asymmetry and cannot be reduced to a mean–variance metric in non-normal settings. In particular, the positive and stable associations observed in core and Northern European bond markets mirror earlier results showing that higher Omega values are linked to favorable, right-skewed distributions in liquid and well-functioning markets.

Despite these contributions, the study has some limitations. It relies on daily yield data, which may not fully capture intraday dynamics or the effects of high-frequency shocks. In addition, while the majority of results are significant, the analysis remains sensitive to data availability and to outliers in certain country-horizon combinations. Future research could extend this framework by using higher-frequency data, exploring alternative thresholds in the Omega calculation, or applying the methodology to other fixed-income segments such as corporate bonds. Another direction would be to investigate the underlying drivers of the observed cross-country heterogeneity, such as liquidity conditions, institutional frameworks, or sovereign risk premia, which were not directly tested in this study. More broadly, the proposed framework offers a ready-to-use methodological tool for evaluating whether the statistical asymmetry of returns, particularly skewness, aligns with the Omega ratio in bond markets, thereby enriching performance assessment in the fixed-income domain.

6- Declarations

6-1- Author Contributions

Conceptualization: A.B. and T.T.; methodology: G.T.; validation: L.P. and T.T.; writing—original draft preparation: L.P. and T.T.; writing—review and editing: G.T. and A.B.; visualization: G.T. and L.P.; supervision: G.T. and A.B. All authors have reviewed and approved the final version of the manuscript for publication.

6-2- Data Availability Statement

Data was obtained from *Investing.com* and are available *https://www.investing.com/rates-bonds/* with the permission of *Investing.com*.

6-3- Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

6-4- Institutional Review Board Statement

Not applicable.

6-5- Informed Consent Statement

Not applicable.

6-6- Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

7- References

[1] Egan, W. J. (2011). The Distribution of S&P 500 Index Returns. SSRN Electronic Journal, 1-15. doi:10.2139/ssrn.955639.

[2] Boyer, M. M., Boyer, T. J., & Sanford, A. (2026). Portfolio size, portfolio composition, and the skewness of returns. Journal of Banking & Finance, 107725. doi:10.1016/j.jbankfin.2026.107725.

[3] Szabó, T., Gáspár, S., & Hegedűs, S. (2025). Development of Financial Indicator Set for Automotive Stock Performance Prediction Using Adaptive Neuro-Fuzzy Inference System. Journal of Risk and Financial Management, 18(8), 435. doi:10.3390/jrfm18080435.

[4] Fong, W. M. (2016). Stochastic dominance and the omega ratio. Finance Research Letters, 17, 7–9. doi:10.1016/j.frl.2015.10.026.

[5] Metel, M. R., Pirvu, T. A., & Wong, J. (2017). Risk management under omega measure. Risks, 5(2), 27. doi:10.3390/risks5020027.

[6] Bernard, C., Caporin, M., Maillet, B., & Zhang, X. (2023). Omega Compatibility: A Meta-analysis. Computational Economics, 62(2), 493–526. doi:10.1007/s10614-022-10306-x.

[7] Van Dyk, F., Van Vuuren, G., & Heymans, A. (2014). Hedge Fund Performance Evaluation Using the Sharpe and Omega Ratios. International Business & Economics Research Journal (IBER), 13(3), 485. doi:10.19030/iber.v13i3.8588.

[8] Kazemi, H., Schneeweis, T., & Gupta, B. (2004). Omega as a performance measure. Journal of Performance Measurement, 8, 16-25.

[9] Bi, H., Huang, R. J., Tzeng, L. Y., & Zhu, W. (2019). Higher-order Omega: A performance index with a decision-theoretic foundation. Journal of Banking and Finance, 100, 43–57. doi:10.1016/j.jbankfin.2018.12.013.

[10] Castro, J. G., Tito, E. A. H., Brandão, L. E. T., & Gomes, L. L. (2019). Crypto-assets portfolio optimization under the omega measure. The Engineering Economist, 65(2), 114–134. doi:10.1080/0013791X.2019.1668098.

[11] Sharma, A., & Mehra, A. (2017). Extended omega ratio optimization for risk-averse investors. International Transactions in Operational Research, 24(3), 485–506. doi:10.1111/itor.12184.

[12] Benhamou, E., Guez, B., & Paris, N. (2019). Omega and Sharpe ratio. arXiv Preprint, arXiv:1911.10254. doi:10.48550/arXiv.1911.10254.

[13] Stoupos, N., & Kiohos, A. (2022). Bond markets integration in the EU: New empirical evidence from the Eastern non-euro member-states. North American Journal of Economics and Finance, 63, 101827. doi:10.1016/j.najef.2022.101827.

[14] Chtourou, H. (2015). Analysis of the European Government Bonds and Debt after the European Financial Crisis. Journal of Centrum Cathedra: The Business and Economics Research Journal, 8(2), 146–164. doi:10.1108/jcc-08-02-2015-b004.

[15] Fernández-Gallardo, Á., & Payá, I. (2025). Public debt burden and crisis severity. European Economic Review, 176, 105028. doi:10.1016/j.euroecorev.2025.105028.

[16] Liu, X., Hu, Y., Liu, F., & Zhou, R. (2025). Monetary Policy and Liquidity of the Bond Market—Evidence from the Chinese Local Government Bond Market. Mathematics, 13(16), 2586. doi:10.3390/math13162586.

[17] Petlele, O., & Buthelezi, E. M. (2025). Shocks of government bonds and yield impact economic growth in South Africa. Cogent Economics & Finance, 13(1), 2448219. doi:10.1080/23322039.2024.2448219.

[18] Randl, O., Simion, G., & Zechner, J. (2025). Pricing and constructing international government bond portfolios. Journal of Financial Economics, 173, 104152. doi:10.1016/j.jfineco.2025.104152.

[19] Moodley, F., Ferreira-Schenk, S., & Matlhaku, K. (2024). Effect of Market-Wide Investor Sentiment on South African Government Bond Indices of Varying Maturities under Changing Market Conditions. Economies, 12(10), 265. doi:10.3390/economies12100265.

[20] Nneka, U. J., Ngong, C. A., Ugoada, O. A., & Onwumere, J. U. J. (2025). Effect of bond market development on economic growth of selected developing countries. Journal of Economic and Administrative Sciences, 41(1), 132–148. doi:10.1108/JEAS-01-2022-0015.

[21] Crespo Cuaresma, J., & Fernández, O. (2024). Explaining long-term bond yields synchronization dynamics in Europe. Economic Modelling, 133, 106684. doi:10.1016/j.econmod.2024.106684.

[22] Balli, F. (2009). Spillover effects on government bond yields in euro zone. Does full financial integration exist in European government bond markets? Journal of Economics and Finance, 33(4), 331–363. doi:10.1007/s12197-008-9029-3.

[23] Barrios, S., Iversen, P., Lewandowska, M., & Setzer, R. (2009). Determinants of intra-euro area government bond spreads during the financial crisis. Directorate General Economic and Financial Affairs (DG ECFIN), European Commission, Brussels, Belgium.

[24] Eijffinger, S. C. W., & Pieterse-Bloem, M. (2023). Eurozone government bond spreads: A tale of different ECB policy regimes. Journal of International Money and Finance, 139, 102965. doi:10.1016/j.jimonfin.2023.102965.

[25] Gabrisch, H., & Orlowski, L. T. (2010). Interest rate convergence in euro-candidate countries: Volatility dynamics of sovereign bond yields. Emerging Markets Finance and Trade, 46(6), 69–85. doi:10.2753/REE1540-496X460605.

[26] Bernoth, K., Von Hagen, J., & Schuknecht, L. (2012). Sovereign risk premiums in the European government bond market. Journal of International Money and Finance, 31(5), 975–995. doi:10.1016/j.jimonfin.2011.12.006.

[27] Abad, P., & Chuliá, H. (2013). European government bond markets and monetary policy surprises: Returns, volatility and integration. IREA Working Papers (No. 201325), Institut de Recerca en Economia Aplicada Regional y Pública, Universitat de Barcelona, Spain.

[28] Warin, T., & Stojkov, A. (2023). Sovereign Bond Yield Differentials across Europe: A Structural Entropy Perspective. Entropy, 25(4), 630. doi:10.3390/e25040630.

[29] Afonso, A. (2010). Long-term government bond yields and economic forecasts: Evidence for the EU. Applied Economics Letters, 17(15), 1437–1441. doi:10.1080/13504850903049627.

[30] Lozada, G. A. (2015). For Constant-Duration or Constant-Maturity Bond Portfolios, Initial Yield Forecasts Return Best near Twice Duration. College of Social and Behavioral Science, The University of Utah, Utah, United States

[31] Keating, C., & Shadwick, W. F. (2002). A universal performance measure. Journal of Performance Measurement, 6(3), 59-84.

[32] Li, Q., & Xie, X. (2024). Worst-case Omega ratio under distribution uncertainty with its application in robust portfolio selection. Probability in the Engineering and Informational Sciences, 38(2), 318–340. doi:10.1017/S0269964823000141.

[33] Kapsos, M., Christofides, N., & Rustem, B. (2014). Worst-case robust Omega ratio. European Journal of Operational Research, 234(2), 499–507. doi:10.1016/j.ejor.2013.04.025.

[34] Zivot, E., & Wang, J. (2003). Rolling Analysis of Time Series. In: Modeling Financial Time Series with S-Plus®. Springer, New York, United States. doi:10.1007/978-0-387-21763-5_9.

[35] Balder, S., & Schweizer, N. (2017). Risk aversion vs. the Omega ratio: Consistency results. Finance Research Letters, 21, 78–84. doi:10.1016/j.frl.2016.12.012.

[36] Croux, C., & Dehon, C. (2010). Influence functions of the Spearman and Kendall correlation measures. Statistical Methods & Applications, 19(4), 497–515. doi:10.1007/s10260-010-0142-z.

[37] Abdi, H., & Williams, L. J. (2010). Principal component analysis. Wiley Interdisciplinary Reviews: Computational Statistics, 2(4), 433–459. doi:10.1002/wics.101.