Available online at www.ijournalse.org

Emerging Science Journal

(ISSN: 2610-9182)

Vol. 10, No. 3, June, 2026

Nonparametric Mixed Moving Average-Extended Exponentially Weighted Moving Average Signed-Rank Control Chart

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Abstract

Control charts are strong statistical monitoring instruments extensively utilized in both manufacturing and non-manufacturing operations. In numerous ongoing processes, the concept of normality is challenging to achieve, resulting in erroneous evaluations within parametric monitoring systems. When the actual variability of a performance parameter is unknown, nonparametric control charts provide a reliable and adaptable approach to evaluating the process. The benefits of utilizing combination control charts encompass increased sensitivity, thorough monitoring, and the capacity to adapt by altering mixtures to satisfy workflow and company needs. To overcome this limitation, this work introduces a hybrid moving average-extended exponentially weighted moving average control chart utilizing the Wilcoxon signed-rank statistic, namely, the MA-EEWMA-WSR, to identify shifts in the process mean. A Monte Carlo simulation was used to estimate the average run length, and the average extra-quadratic loss (AEQL) was calculated to comprehensively assess the performance of the control chart for some selected symmetric distributions: normal, Laplace, and logistic. The study shows that the proposed technique demonstrated efficacy in detecting all alterations across various distributions, outperforming other charts, such as MA-EEWMA, EWMA-WSR, and EEWMA-WSR under the zero-state scenario. The efficiency of the proposed chart in identifying process adjustments is demonstrated in a case study of the dry bleach products dataset.

Keywords:

Nonparametric Control Chart;
Wilcoxon Signed-Rank Test;
Average Run Length;
Extended EWMA;
Moving Average.

Article History:

Received:	27	December	2025
Revised:	09	May	2026
Accepted:	11	May	2026
Published:	01	June	2026

1- Introduction

Statistical Process Control (SPC) is an innovative, data-centric approach that uses statistical techniques to monitor, control, and improve a production or operational process. The primary objective of SPC is to ensure that a process functions efficiently and consistently over time, resulting in reduced waste, lower costs, and improved product quality and customer satisfaction. SPC prioritizes early identification and prevention of issues rather than rectifying flaws after they occur. A core principle of SPC is the recognition that all processes demonstrate variance. SPC distinguishes between two fundamental categories of variation: (i) Common Cause Variation (or Random Variation): This refers to the intrinsic, natural, and foreseeable variability present within a stable process. It arises from many persistent elements that are anticipated components of the process. A method that functions solely with common cause variation is considered to be in control or stable. (ii) Special Cause Variation (or Assignable Cause Variation): This is an unanticipated variation resulting from external, sporadic, or important causes that are not often included in the procedure. It indicated that the process was out of control. The Control Chart (also known as a Shewhart Chart), invented by Shewhart [1], is the fundamental instrument in SPC. It is a graphical instrument that simplifies the monitoring of a process's performance and distinguishes between common and significant cause variation, making it optimal for detecting major shifts. Later,

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DOI: https://doi.org/10.28991/ESJ-2026-010-03-05

some authors developed control charts to detect minor variations, such as CUSUM [2], the exponentially weighted moving average (EWMA) [3], and the moving average (MA) [4] control charts. Nonetheless, many authors have devised novel control charts that build on traditional ones, including the double exponentially weighted moving average (DEWMA) control chart [5], the double moving average control chart (DMA) [6], the extended exponentially weighted moving average (EEWMA) [7], the modified exponentially weighted moving average control chart (MEWMA) [8], and the triple moving average (TMA) control chart [9]. A single control chart is universally preferable for spotting changes. This indicates that many researchers have created innovative control charts to detect deviations and immediately return the process to normalcy.

One way to improve process detection efficiency is to create a combination control chart that better identifies rapid changes. This represents the concept of combined control charts, also called mixed control charts. Numerous authors have developed mixed control charts. For example, in detecting minor and moderate shifts, the EWMA-CUSUM, first introduced by Abbas et al. [10] and then refined in 2015 by Zaman et al. [11] for the mixed CUSUM-EWMA chart, shows better capability and efficiency. In 2017, Aslam et al. [12] developed a new chart that outperforms traditional control charts by integrating DMA and EWMA to evaluate dependability under skew distributions. According to Ali & Hag [13], the generally weighted moving average (GWMA) chart combined with the CUSUM chart is superior to other charts for detecting small changes in the operating mean. Lu & Lin [14] presented a novel MA-Exponentiated EWMA control chart to enhance the identification of minor shifts and incremental variations. The findings indicate that this strategy is particularly effective for monitoring processes characterized by roughly symmetric distributions and minor magnitude shifts. Saesuntia et al. [15] investigated integrating MA with Triple EWMA (TEWMA) control charts to effectively detect changes. This study concludes that the mixed control chart (TEWMA-MA) outperforms other charts for detecting small to moderate variations. Recently, Ahsan et al. [16] presented an improved EWMA Max-Multivariate (EWMA Max-M) control chart to facilitate quick identification of process abnormalities. The suggested approach provides superior performance in identifying minor shifts, also evidenced by ARL evaluations. All review findings demonstrate that mixed control charts can adapt rapidly to changes and are applicable across multiple situations.

A common hypothesis used when building control charts is that the quality attribute in concern is a normally distributed random variable. Nevertheless, in numerous instances, this presumption is contravened, thereby rendering the conventional parametric control chart potentially useless in assessing the process. As a result, numerous authors have developed nonparametric control charts. The sign and signed-rank statistic are nonparametric statistics used to detect changes in the location parameter after the desired value has been determined. Especially, the signed-rank statistic necessitates the supplementary requirement of symmetry. The signed-rank statistic has served as the basis for numerous control charts published by various authors. Graham et al. [17] presented an EWMA control chart that included a nonparametric signed-rank statistic, and reported that a comprehensive simulation analysis was conducted to evaluate its performance relative to existing control charts. In comparison with the control charts for GWMA-SN and EWMA-SN, a GWMA-SR based on a signed-rank control chart created by Sukparungsee [18] showed better robustness for monitoring skew process shifts. A ranked set sampling method for process location monitoring utilizing a CUSUM-Wilcoxon signed-rank chart, as suggested by Abid et al. [19].

Overall, the suggested chart surpasses certain competitor charts in shift detection, as indicated by the average run length. Using Sign and Signed-rank statistics, nonparametric moving average control charts were provided by Pawar et al. [20] to identify changes in the median among symmetrical procedure models. A triple exponentially weighted moving average, which is a nonparametric Wilcoxon signed-rank control chart, was developed by Rasheed et al. [21], using ranked set sampling as its basis. Additionally, their results demonstrate that the proposed chart outperforms the current control charts in detecting small changes in the operating location. The NPPM-SR, a nonparametric progressive mean control chart based on the Wilcoxon signed-rank statistic, was developed by Abbas et al. [22] and outperforms the NPEWMA-SN and NPEWMA-SR charts, which both use average run length criteria, in terms of analysis efficiency. In order to detect changes in the mean parameter, Petcharat & Sukparungsee [23] provide a MEWMA-Wilcoxon Signed Rank chart.

Talordphop et al. [24] introduced the EEWMA, adopting signed-rank statistics for tracking the procedure mean, and found that the designed chart exhibits superior efficacy in detecting shifts. A mixed EWMA-MA based on the signed-rank statistic was introduced by Raza et al. [25], who used run-length properties to assess the effectiveness of control charts. Using the Signed-rank statistic, a double Homogeneously Weighted Moving Average control chart for enhancing quality control in an industrial process was proposed by Abid et al. [26]. According to Alevizakos [27], a double Homogeneously Weighted Moving Average Signed-rank control chart (DHWMA-SR) was developed in 2025 for the purpose of monitoring the location parameter. The run length properties indicate that the proposed chart performs better in the zero-stage setting, but it has poor steady-state performance. Abbas et al. [28] present the EEWMA chart derived from the Wilcoxon signed rank test with ranked set sampling, and the outcome assessment indicates that the suggested chart is acceptable. Most outcomes of the review demonstrate that nonparametric control charts may rapidly adapt to changes and be employed in various settings with few constraints.

The advantages of employing combined control charts include enhanced sensitivity, comprehensive monitoring, and the ability to adjust by modifying mixes to meet workflow and business requirements. The MA control chart layout is simpler than that of EWMA and EEWMA charts; however, it is less effective in detecting subtle process changes. The EEWMA control chart provides an improved framework based on EWMA and is highly effective in detecting subtle process alterations. Thus, the integrated MA-EEWMA can improve the surveillance of the process mean. This research integrates two effective charts (quick detection and user-friendliness in non-normal process scenarios) to identify changes promptly. The MA-EEWMA, which applies the signed-rank statistic, was introduced to improve the detection of alterations in the procedure-average parameter. The application of the Monte Carlo simulation to determine the average run length (ARL) enabled us to evaluate the effectiveness of the control charts. Finally, we implement the proposed chart in an actual-life scenario and compare it with existing control charts to demonstrate its practical significance.

2- Design of Control Charts

Assume $X_{jk}, j = 1,2,3,\dots; k = 1,2,3,\dots,n$ to be the process variable, independently and identically distributed, chosen from a normal distribution with mean μ and standard deviation σ. The configuration of the control chart is fundamentally as follows.

2-1-Classical Control Chart

MA Chart:

The formula of the MA statistic [4] of period j for each span (ω) is presented below:

$$MA_j = \begin{cases} \frac{\sum_{k=1}^{j} X_j}{j}, \mathrm{j} < \omega \\ \frac{\sum_{k=j-\omega+1}^{j} X_j}{\omega}, \mathrm{j} \geq \omega \end{cases} \tag{1}$$

The upper (UCL) and lower (LCL) controlling constraint boundaries on the MA chart can be determined which specifies H_1 as the measurement coefficients from each of the control limits:

$$UCL/LCL = \begin{cases} \mu \pm \frac{H_1\sigma}{\sqrt{j}}, \mathrm{j} < \omega \\ \mu \pm \frac{H_1\sigma}{\sqrt{\omega}}, \mathrm{k} \geq \omega \end{cases} \tag{2}$$

EWMA chart:

The EWMA statistics using smoothing parameter ζ which vary from 0 to 1 is detail here:

$$Z_j = \zeta X_j + (1-\zeta)Z_{j-1} \tag{3}$$

The control limits of the EWMA graph, which provide H_2 as the coefficients of the control limits, are defined by:

$$UCL/LCL = \mu \pm H_2\sigma\sqrt{\frac{\zeta}{2-\zeta}} \tag{4}$$

EEWMA chart:

The Extended EWMA control chart statistic, using smoothing parameters ζ_1, ζ_2, which vary from 0 to 1, is provided below, with the set $0 < \zeta_1 \leq 1$ and the set $0 \leq \zeta_2 < \zeta_1$:

$$Y_j = \zeta_1 X_j - \zeta_2 X_{j-1} + (1-\zeta_1+\zeta_2)Y_{j-1} \tag{5}$$

The control limits of the Extended EWMA graph, which provide H_3 as the coefficients of the control limits, are defined by:

$$UCL/LCL = \mu \pm H_3\sigma\sqrt{\frac{\zeta_1^2+\zeta_2^2-2\zeta_1\zeta_2(1-\zeta_1+\zeta_2)}{2(\zeta_1-\zeta_2)-(\zeta_1-\zeta_2)^2}} \tag{6}$$

2-2- Wilcoxon signed-rank chart

Suppose $X_{jk}, j = 1,2,3, \ldots; k = 1,2,3, \ldots, n$ to be the process variable and its process mean is the target value α.

Let $R_{jk} = |X_{jk} - \alpha|$ indicate the rank of the absolute difference; thereby, the Wilcoxon signed rank (WSR) statistics is defined here:

$$WSR = \sum I_{jk} R_{jk} \tag{7}$$

where $I_{jk} = \begin{cases} 1; (X_{jk} - \alpha) > 0 \\ 0; (X_{jk} - \alpha) = 0 \\ -1; (X_{jk} - \alpha) < 0 \end{cases}$.

The mean and variance of WSR statistic are 0 and $\big(n(n+1)(2n+1)\big)/6$, respectively.

2-3- Mixed control chart

MA-Extended EWMA Chart:

The mixed MA-Extended EWMA chart [29] is obtained by combining the MA and the Extended EWMA control charts. Here is the definition of the MA-Extended EWMA statistic:

$$ME_j = \begin{cases} \frac{\sum_{k=1}^{j} Y_j}{j}, \text{j} < \omega \\ \frac{\sum_{k=j-\omega+1}^{j} Y_j}{\omega}, \text{j} \geq \omega \end{cases} \tag{8}$$

The expected value is μ and asymptotic variance of the MA-Extended EWMA statistic are given as:

$$V(ME_j) = \begin{cases} \frac{\sigma^2}{j^2}\left[\frac{\zeta_1^2+\zeta_2^2-2\zeta_1\zeta_2(1-\zeta_1+\zeta_2)}{2(\zeta_1-\zeta_2)-(\zeta_1-\zeta_2)^2}\right], \text{j} < \omega \\ \frac{\sigma^2}{\omega^2}\left[\frac{\zeta_1^2+\zeta_2^2-2\zeta_1\zeta_2(1-\zeta_1+\zeta_2)}{2(\zeta_1-\zeta_2)-(\zeta_1-\zeta_2)^2}\right], \text{j} \geq \omega \end{cases} \tag{9}$$

The MA-Extended EWMA chart's control limit boundaries, which provide H_4 as the coefficients of the control limits, are defined by:

$$UCL/LCL = \begin{cases} \mu \pm H_4 \frac{\sigma}{j}\sqrt{\frac{\zeta_1^2+\zeta_2^2-2\zeta_1\zeta_2(1-\zeta_1+\zeta_2)}{2(\zeta_1-\zeta_2)-(\zeta_1-\zeta_2)^2}}, \text{j} < \omega \\ \mu \pm H_4 \frac{\sigma}{\omega}\sqrt{\frac{\zeta_1^2+\zeta_2^2-2\zeta_1\zeta_2(1-\zeta_1+\zeta_2)}{2(\zeta_1-\zeta_2)-(\zeta_1-\zeta_2)^2}}, \text{j} \geq \omega \end{cases} \tag{10}$$

The EWMA-WSR Chart:

The statistical model of the EWMA WSR chart is outlined as:

$$EW_j = \zeta WSR_j + (1-\zeta)EW_{j-1}. \tag{11}$$

The control limits of the EWMA WSR graph, which provide H_5 as the coefficients of the control limits, are defined by:

$$UCL/LCL = \pm H_5\sqrt{\frac{\zeta}{2-\zeta}\frac{n(n+1)(2n+1)}{6}} \tag{12}$$

The EEWMA-WSR Chart:

The statistical model of the EEWMA WSR chart is outlined as:

$$EEW_j = \zeta_1 WSR_j - \zeta_2 WSR_{j-1} + (1-\zeta_1+\zeta_2)EEW_{j-1} \tag{13}$$

The control limits of the Extended EWMA WSR graph, which provide H_6 as the coefficients of the control limits, are defined by

$$UCL/LCL = \pm H_6\sqrt{Var(EEW_j)\frac{n(n+1)(2n+1)}{6}} \tag{14}$$

where $Var(EEW_j) = \frac{\zeta_1^2+\zeta_2^2-2\zeta_1\zeta_2(1-\zeta_1+\zeta_2)}{2(\zeta_1-\zeta_2)-(\zeta_1-\zeta_2)^2}$.

The Proposed Chart:

The MA-EEWMA framework introduces methodological distinctiveness by integrating a double-smoothing structure that employs two sequential layers of memory-based filtering: the initial layer is MA, followed by a smoothing window applied to the EEWMA outputs. The second layer, termed EEWMA, enhances the traditional EWMA framework by employing twin weighting parameters that assign differing levels of significance to present and historical data. The proposed chart integrates the advantages of the mixed MA-EEWMA and Wilcoxon signed-rank control charts, notably the latter's sensitivity to minor deviations. Simultaneously, the other is autonomous from distribution parameters. The proposed chart approach resembles MA-EEWMA, but replaces the mean with zero. The upper and lower control limits do not use the standard deviation; rather, they are determined from the variance of WSR. Let will define the MA-Extended EWMA-WSR statistic as follows:

$$MEW_j = \begin{cases} \frac{\sum_{k=1}^{j} WSR_j}{j}, \text{j} < \omega \\ \frac{\sum_{k=j-\omega+1}^{j} WSR_j}{\omega}, \text{j} \geq \omega \end{cases} \tag{15}$$

This is the asymptotic variance of the MA-Extended EWMA WSR statistic, with an expected value of zero:

$$V(MEW_j) = \begin{cases} \frac{n(n+1)(2n+1)}{6j^2}\left[\frac{\zeta_1^2+\zeta_2^2-2\zeta_1\zeta_2(1-\zeta_1+\zeta_2)}{2(\zeta_1-\zeta_2)-(\zeta_1-\zeta_2)^2}\right], \text{j} < \omega \\ \frac{n(n+1)(2n+1)}{6\omega^2}\left[\frac{\zeta_1^2+\zeta_2^2-2\zeta_1\zeta_2(1-\zeta_1+\zeta_2)}{2(\zeta_1-\zeta_2)-(\zeta_1-\zeta_2)^2}\right], \text{j} \geq \omega \end{cases} \tag{16}$$

The MA-Extended EWMA-WSR chart's control limit boundaries, which provide H_7 as the coefficients of the control limits, are defined by:

$$UCL/LCL = \pm H_7\sqrt{V(MEW_j)} \tag{17}$$

3- Assessment Methodology

This section outlines the performance analysis concerning the zero-state situation, similar to Alevizakos et al. [30] and Karoon & Areepong [31]. We emphasize zero-state performance as it most effectively assesses and identifies shifts promptly from the initiation of monitoring, especially for short-run processes. Nonetheless, for sustained output, steady-state conditions offer a more dependable assessment of performance. Prevalent practice to evaluate a control chart's efficacy using the run length distribution and related metrics. A critical performance indicator, the average run length (ARL), is the average of all the tracked sightings on a control chart before the first out-of-control signal is triggered. Academic literature has presented numerous methodologies for ARL assessment [32-34]. An effective control chart will have a big ARL_0 (in control) to prevent false alarms and a tiny ARL_1 (out of control) to detect shifts fast. Typically, for several charts, the steady-state ARL is larger than the zero-state ARL, indicating the time required for the chart's memory to adjust to the process's new state. The ARL is define below as follows:

$$ARL = \frac{\sum_{j=1}^{N} RL_j}{N} \tag{18}$$

where, RL_j signifies the count of testing necessary till the method turns impossible within the starting period.

To evaluate the precision of control charts, the fundamental estimation method uses Monte Carlo simulation to generate computational results. Assume the specified values in the simulation are $n = 5$, $ARL_0 = 370$, $\omega = 5$, and the smoothing parameter is $\zeta_1 = 0.1, \zeta_2 = 0.03$. Another organization of simulation details is presented below. Beginning with, produce a random sample from symmetric distributions. Subsequently, calculate the control chart statistics, the control limit coefficients, and the control limit borders. Finally, calculate the ARL_1 for 50,000 rounds. To obtain results, according to the procedure illustrated in Figure 1.

To determine the overall efficacy of this control diagram, we use the average extra-quadratic loss (AEQL) [35]. The AEQL represents the average weighted value of the ARL computed for several shifts within a process. It serves as the benchmark for evaluating several mixed charts to determine which is more "robust" when the precise shift size is indeterminate in advance. An improved control chart would exhibit lower AEQL scores, indicating better results.

$$AEQL = \frac{1}{\delta_{Max}-\delta_{Min}}\sum_{\delta=0}^{\delta_{Max}} \delta^2 ARL(\delta) \tag{19}$$

where $ARL(\delta)$ represents the ARL depicted on a chart associated with the designated shift. δ shows the size of the shift that happens throughout the procedure. δ_{Max} and δ_{Min} are the maximum and minimum shifts evaluated through the process, respectively.

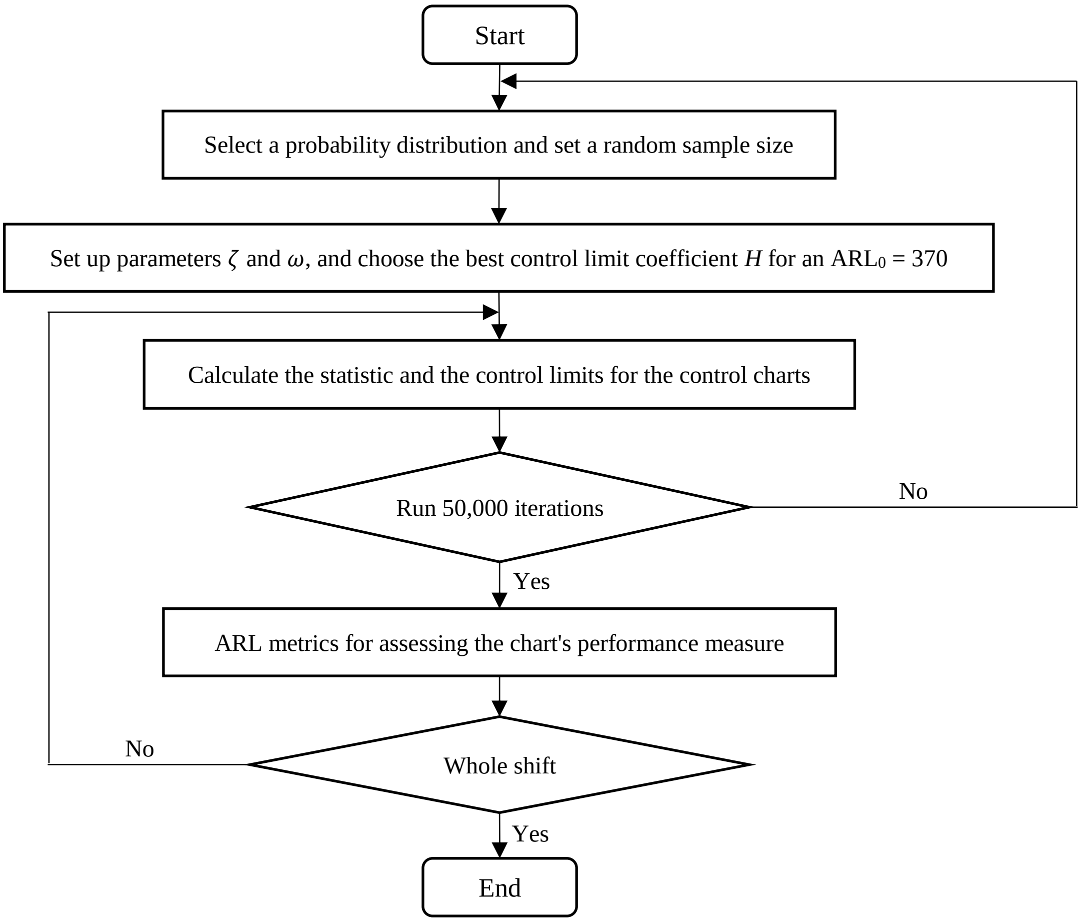

Figure 1. A flow diagram showing the procedure for establishing performance metrics

4- Result

4-1- Comparative Study

The following section clarifies the experimental investigation and evaluates the effectiveness of the suggested chart in comparison to the EWMA-WSR and EEWMA-WSR control charts under $ARL_0 = 370$ in symmetric distributions. The visualization showing the minimum ARL_1 value during a particular shift is optimal.

Tables 1 to 3 presents the comparison of the ARL_1 value for the proposed chart with other charts for various distributions. In the zero-state situation, the MA-EEWMA-WSR demonstrates the highest efficacy across all distributions for the range of shifts, compared with MA-EEWMA, EWMA-WSR and EEWMA-WSR charts. In addition, the AEQL value, which reflects the overall effectiveness of the proposed chart, is significantly lower than that of earlier charts, indicating that the suggested chart surpasses its predecessors. Figure 2 provides the overall AEQL values according to distributions.

Table 1. The ARL_1 results in the comparison research chart follow the normal distribution

Shift	MA-EEWMA $H_4 = 2.12$	EWMA-WSR $H_5 = 8.65$	EEWMA-WSR $H_6 = 12.24$	MA-EEWMA-WSR $H_7 = 6.90$
0	370.71	370.14	370.53	370.65
0.05	320.14	242.66	238.42	197.06
0.1	229.17	174.89	174.54	104.9
0.25	78.56	101.53	100.3	51.54
0.5	26.22	81.48	94.09	21.98
0.75	14.33	59.64	73.32	8.79
1	9.84	61.81	47.44	3.69
1.5	6.14	59.51	45.39	2.53
AEQL	4.63	25.83	22.29	2.46

Table 2. The ARL_1 results in the comparison research chart follow the Laplace distribution

Shift	MA-EEWMA $H_4 = 2.22$	EWMA-WSR $H_5 = 8.79$	EEWMA-WSR $H_6 = 15.04$	MA-EEWMA-WSR $H_7 = 5.79$
0	370.59	370.46	370.56	370.67
0.05	349.35	325.44	312.36	**157**
0.1	293.18	282.16	237.93	**99.83**
0.25	139.37	223.07	116.59	**65.87**
0.5	48.55	52.25	103.53	**36.12**
0.75	25.17	49.68	100.54	**18.75**
1	16.21	48.38	95.07	**11.86**
1.5	9.41	43.49	90.6	**3.97**
AEQL	7.62	20.48	39.18	**4.59**

Table 3. The ARL_1 results in the comparison research chart follow the Logistic distribution

Shift	MA-EEWMA $H_4 = 12.34$	EWMA-WSR $H_5 = 4.96$	EEWMA-WSR $H_6 = 11.55$	MA-EEWMA-WSR $H_7 = 4.91$
0	370.67	370.32	370.51	370.59
0.05	346.17	172.63	167.62	155.32
0.1	306.34	100.21	94.17	92.28
0.25	252.51	46.41	42.43	40.01
0.5	209.23	16.73	20.22	16.24
0.75	161.48	13.12	16.55	10.9
1	105.6	10.43	13.63	8.79
1.5	59.63	9.42	11.83	7.64
AEQL	40.26	4.75	5.86	4

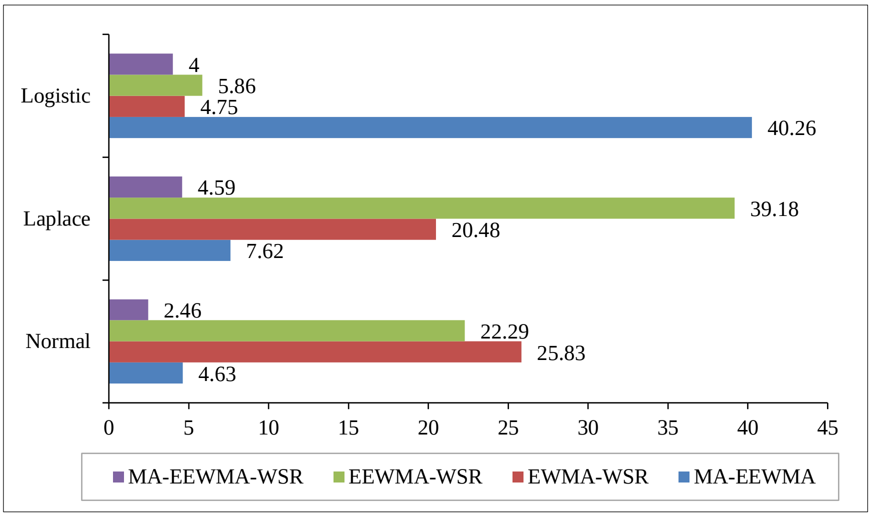

Figure 2. AEQL value displays for control charts within distributions

Within the framework of normal distribution, it has been noted that the calculated ARL_1values from the MA-EEWMA-WSR control chart are lower compared with those of the MA-EEWMA, EWMA-WSR, and EEWMA-WSR charts across all shift magnitudes, clearly demonstrated in Table 1.

The analysis of the Laplace distribution produces findings that align with the Logistic distribution, demonstrating that the proposed chart outperforms the MA-EEWMA, EWMA-WSR, and EEWMA-WSR charts across all shifts (0-1.5), as illustrated in Tables 2 and 3.

Tables 4 to 6 shows the values of ARL_1 in the proposed chart vary parameter ω and n based on normal (0,1), Laplace(0,1), and Logistic(6,2) distributions. It is also observed that the value of ARL_1 decreases when the span ωincreases, matching the outcome found with n.

Table 4. **The ARL_1 results in the proposed chart follow the normal distribution vary n and ω**

Shift	$n = 5$		$n = 10$	
	$\omega = 5$	$\omega = 10$	$\omega = 5$	$\omega = 10$
0	370.65	370.51	370.49	370.55
0.05	197.06	193.32	195.26	192.11
0.1	104.9	101.07	101.87	99.85
0.25	51.54	48.83	48.22	45.9
0.5	21.98	20.13	18.6	18.42
0.75	8.79	8.41	8.4	8.39
1	3.69	3.55	3.71	3.72
1.5	2.53	2.5	2.55	2.56

Table 5. **The ARL_1 results in the proposed chart follow the Laplace distribution vary n and ω**

Shift	$n = 5$		$n = 10$	
	$\omega = 5$	$\omega = 10$	$\omega = 5$	$\omega = 10$
0	370.67	370.53	370.61	370.58
0.05	157	154.25	155.28	152.99
0.1	99.83	96.51	97.7	95.22
0.25	65.87	62.89	63.31	60.03
0.5	36.12	33.34	34.45	31.16
0.75	18.75	16.23	16.1	15.46
1	11.86	11.8	11.88	11.89
1.5	3.97	3.91	3.99	3.93

Table 6. **The ARL_1 results in the proposed chart follow the Logistic distribution vary n and ω**

Shift	$n = 5$		$n = 10$	
	$\omega = 5$	$\omega = 10$	$\omega = 5$	$\omega = 10$
0	370.59	370.6	370.62	370.59
0.05	155.32	153.11	153.18	152.25
0.1	92.28	90.04	89.67	88.44
0.25	40.01	38.84	37.79	36.65
0.5	16.24	14.42	14.52	13.93
0.75	10.9	9.02	9.03	8.99
1	8.79	8.7	8.75	8.71
1.5	7.64	7.61	7.69	7.67

The details are shown below:

MA-EEWMA-WSR compared to MA-EEWMA:

The first foundation of the MA-EEWMA idea was [29]. The proposed method outperforms the MA-EEWMA control chart in all three distribution types (normal, Laplace, and Logistic). The AEQL values of the proposed MA-EEWMA-WSR for normal, Laplace, and Logistic distributions are 2.46,4.59,4.00, while the MA-EEWMA of AEQL = 4.63, 7.62, 40.26, respectively. The performance of the proposed chart is better than that of the MA-EEWMA chart.

MA-EEWMA-WSR compared to EWMA-WSR:

The concept for EWMA-WSR first appeared in Graham et al. [17]. This study's suggested technique exhibits superior efficacy relative with a EWMA-WSR control chart spanning normal, Laplace, and Logistic distributions. The AEQL scores associated with MA-EEWMA-WSR and EWMA-WSR across normal, Laplace, and Logistic distributions are (25.83, 2.46), (20.48, 4.59), and (4.75, 4.00), respectively, as shown in Tables 1 to 3. Therefore, when compared to the EWMA-WSR chart, the suggested chart performs far better.

MA-EEWMA-WSR compared to EEWMA-WSR:

The original source of the concept of EEWMA-WSR was [24]. The proposed method outperforms the EEWMA-WSR control chart in all three distribution types (normal, Laplace, and Logistic). The AEQL values of the proposed

MA-EEWMA-WSR for normal, Laplace, and Logistic distributions are 2.46,4.59,4.00, while the EEWMA-WSR of AEQL = 22.29,39.18,5.86, respectively. The performance of the proposed chart is better than that of the EEWMA-WSR chart.

The comparison analysis assesses the ARL efficacy of the proposed chart relative to the mixed EWMA-MA [35] and DHWMA [9, 26] using the Wilcoxon signed-rank control chart. This simulation study suggested that a mixed control chart utilizing the Wilcoxon signed-rank statistic can similarly facilitate the rapid detection of a shift in the process mean.

4-2- Real-Life Example

We use the net weight (in oz) of a dataset on dry bleach products, provided by Montgomery [36]. For each sampling site, 20 samples, which consist of $n = 5$, are gathered to identify a shift in the mean of a dry bleach product. The process has a reported mean of 16.27, and the Shapiro-Wilk goodness-of-fit test yields a p-value of 0.37. Consequently, the dataset has a symmetric distribution. We construct the MA-EEWMA, EWMA-WSR, EEWMA-WSR, and MA-EEWMA-WSR charts. The control charts are displayed in Figure 3 to 6 and describe that the proposed MA-EEWMA-WSR chart gives an out-of-control signal at the first sample, the MA-EEWMA can detect at the second sample, while the EWMA-WSR and EEWMA-WSR do not detect any shifts. The present instance shows that the suggested chart performs better than the others. To choose a parameter, investigators generally use an initial parameter set informed by previous studies. The smoothing parameter of EEWMA varies from 0.05 to 0.2, whereas the span parameter typically runs from 3 to 7. In practice, the span parameter is often selected depending on the anticipated duration of a suitable cause or the usual number of samples collected during a manufacturing shift.

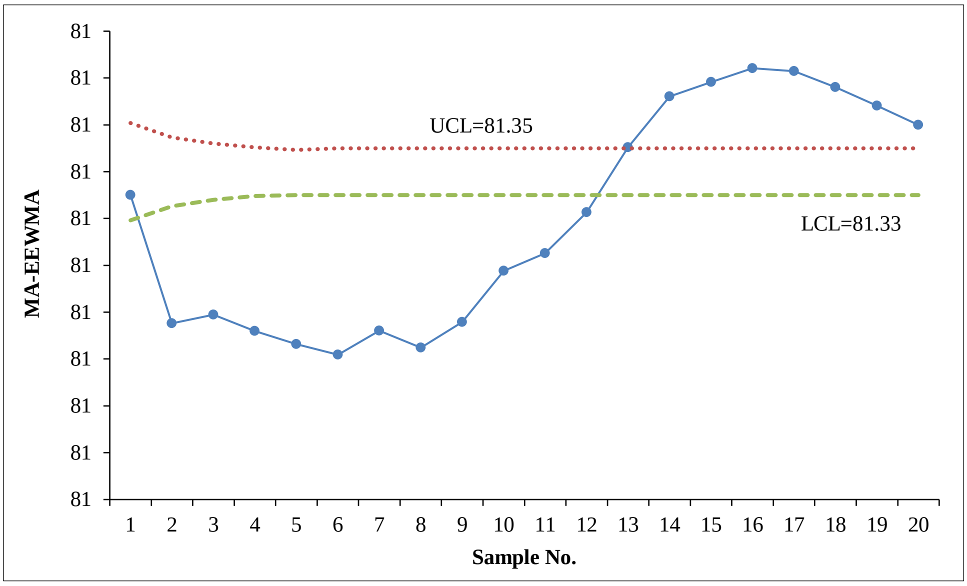

Figure 3. A control chart for the MA-EEWMA dataset containing dry bleach products

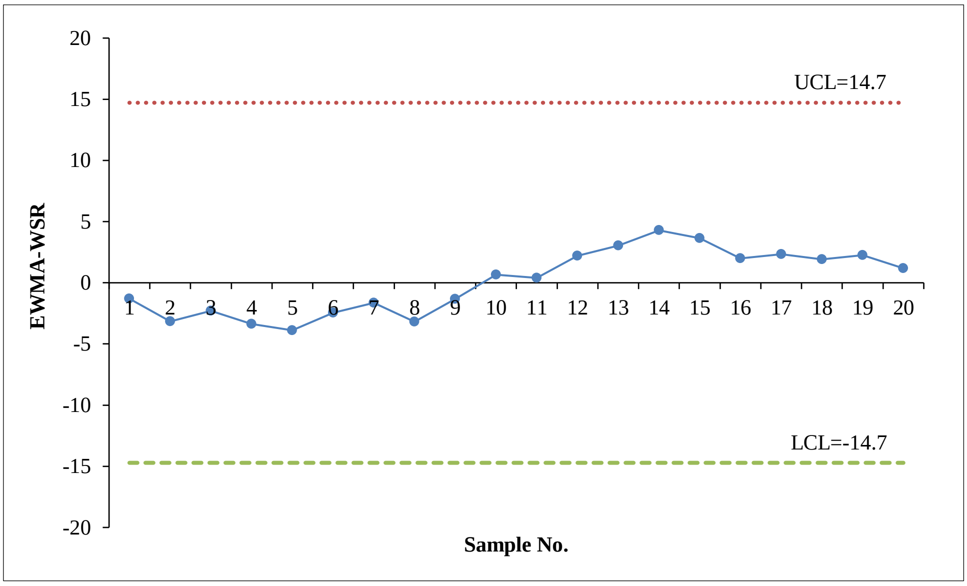

Figure 4. A control chart for the EWMA-WSR dataset containing dry bleach products

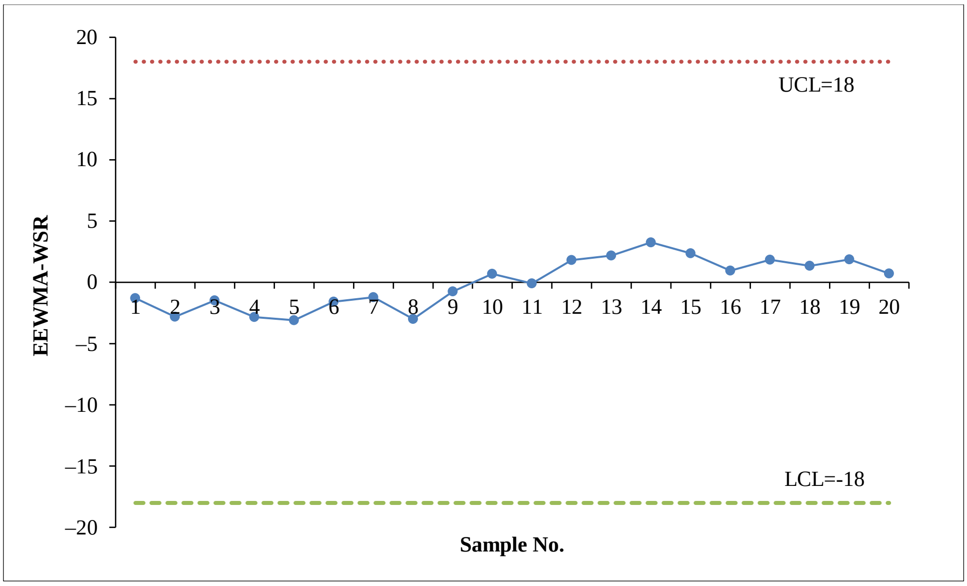

Figure 5. A control chart for the EEWMA-WSR dataset containing dry bleach products

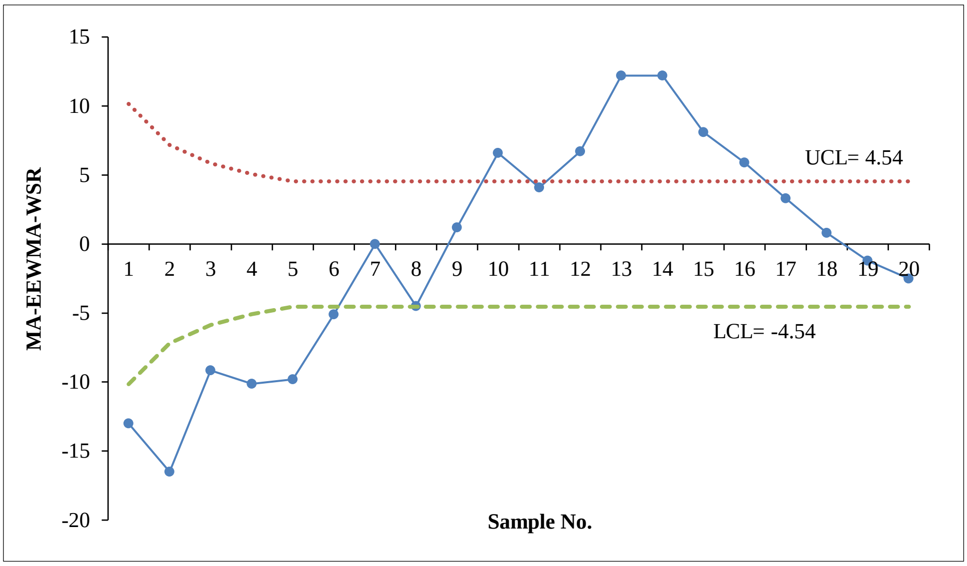

Figure 6. A control chart for the MA-EEWMA-WSR dataset containing dry bleach products

5- Conclusion

Control charts are robust statistical monitoring tools employed extensively throughout a variety of processes, both industrial and in other industries. In many continuing processes, the notion of normality is difficult to meet, leading to incorrect assessments within parametric monitoring frameworks. For process evaluations where the true variability of a performance measure is unknown, nonparametric control charts provide a reliable and flexible option. The positive effects of deploying combination control charts contain greater reactivity to little and moderate shifts, independence in customization, complete monitoring, and the capacity to adapt through changing blends to fulfill operational demands. This study combines two efficient charts (rapid detection and accessibility in unusual process settings) to immediately recognize changes.

A mixed MA-EEWMA control chart based on the Wilcoxon signed-rank statistic was presented for detecting shifts in the process mean. The average run length (ARL) evaluates the efficiency of control charts, whereas the average extra-quadratic loss (AEQL) assesses the overall effectiveness of the control diagram. A small ARL_1 identifies that the control chart can detect shifts rapidly. The overall results describe that the suggested method showed effectiveness in identifying every change across all distributions. By comparing the suggested chart to other control charts, a specific case study shows how well it identifies process adjustments. The results of this study are similar to previous research, which suggests that a mixed control chart using the Wilcoxon signed-rank statistic can facilitate rapid detection of a shift in the process mean. However, the robustness of signed-rank statistics works well for skewed, heavy-tailed, or unknown distributions, and it demonstrates greater power than sign charts for symmetric distributions. In future research, we plan to expand this methodology to generate non-symmetric distributions and implement them in real applications.

6- Declarations

6-1- Author Contributions

Conceptualization, S.S. and K.T.; methodology, K.T.; software, K.T.; validation, K.T. and S.S.; formal analysis, S.S.; investigation, K.T.; resources, K.T.; data curation, K.T.; writing—original draft preparation, K.T.; writing—review and editing, S.S.; visualization, K.T. and S.S.; supervision, K.T.; project administration, K.T.; funding acquisition, K.T. All authors have read and agreed to the published version of the manuscript.

6-2- Data Availability Statement

The data presented in this study are openly available in Montgomery [36] collected the dataset of dry bleach products.

6-3- Funding

We would like to express our gratitude to National Research Council of Thailand (NRCT) for supporting the research fund with Contract Number N42A670854.

6-4- Acknowledgments

The authors would like to express the appreciation to Rajamangala University of Technology Lanna for their all support.

6-5- Institutional Review Board Statement

Not applicable.

6-6- Informed Consent Statement

Not applicable.

6-7- Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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