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Modeling Contaminant Transport of Nitrate in Soil Column

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Abstract

Soil forms solution when it is in contact with water or any liquid. This soil solution disperses into the ground in different parts, at different velocities. Hence, the chemical contents of the soil are leached gradually from soil with infiltrating water. Soil parameter characterizing this phenomenon is referred to as Solute dispersivity. The objective of this study is to model contaminant transport of nitrate in soil, calibrate and verify the model derived. Dispersion studies were performed in the laboratory using soil columns filled with silver nitrate (AgNO₃) solution. Samples were collected from the column outlet at intervals of 5minutes and the dispersion coefficient calculated. The dispersion coefficient calculated was incorporated into existing Notordamojo's model and solved. Results obtained from the research showed that the R² values ranging from 0.741 to 0.896 and 0.484 to 0.769 were obtained for the modified model and the existing Notordamojo's model respectively. The model verified with the experimental data showed predicted transport was in close agreement with experimental values having coefficient of correlation (r) ranging from 0.86 to 0.98. The difference between the experimental and predicted results, when expressed as a percentage of the experimental value was less than 5%. The study has established that the modified model which accounted for variability in dispersion coefficient offered a better approach than the conventional one.

Keywords:

Silver Nitrate; Concentration; Transport; Dispersion; Soil; Column.

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1- Introduction

With the increasing population growth and the industrial and agricultural development, the amounts of wastes generated are increasing day by day in Nigeria thereby polluting the air, water and soil which becomes a threat to human health. The increasing solid waste and their disposal sites like landfills are not only causing groundwater contamination and soil pollution but also leaving many productive lands as waste lands [1]. It is also possible for untreated waste from septic tanks and toxic chemicals from underground storage tanks to contaminate groundwater. Therefore, there is a need to reduce and recycle of waste. Leachates generated from the wastes contains different types of harmful chemical, toxins and harmful metals. Hence, if the leachate is not properly regulated, it may migrate through the landfill liner and pollute the groundwater. Solute transport in soil and groundwater is affected by a large number of physical, chemical and microbial processes and media properties [2]. Poor microbial quality of groundwater systems has caused outbreaks of diseases in some communities. Where the source of drinking water is shallow groundwater, there may be a risk of contamination from septic tank effluents. Nitrate pollution below croplands is a long-term legacy [3, 4], Hence, is considered as the most widespread inorganic pollutant around the world [5, 6]. Displacement studies such as sorption, adsorption, advection and retardation are important tools for understanding transport of solutes through the soil. These

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studies provide insight into contaminant transport processes such as diffusion sorption, retardation and dispersion [7-9]. Transport of nitrate through the soil is a major source of groundwater contamination. Consumption of groundwater with high concentrations can lead to death.

Zaheer et al. (2017) and Zhang et al. (2016) focused on an experimental study on solute transport in one-dimensional clay soil columns. The study examined Advection-Dispersion Equation (ADE), Two-Region Model (TRM), Continuous Time Random Walk (CTRW), and Fractional Advection-Dispersion Equation (FADE). It was found that all the models can fit the transport process very well; however, ADE and TRM were somewhat unable to characterize the transport behavior in leaching. The CTRW and FADE models were better in capturing the full evaluation of tracer-breakthrough curve and late –time tailing in leaching [10, 11].

Yu et al. (2018), numerically investigated the transformation and transport of contaminant through layered soil with large deformation. This study considered soil weight, sorption and biodegradation. Model validation and applications are achieved through case studies of double-layered finite soil, with the transport and transformation process of contaminant being reproduced numerically. It was found that the breakthrough time of contaminant obtained from the linear adsorption solution is greater than the case of non-linear adsorption solution, which can provide a reference for the design of landfill liner [12].

Modeling of contaminant transport during an urban pluvial flood event was studied by Sämann et al. (2019) [13]. In their study, A Lagrangian particle based transport model is introduced to calculate the potential contamination paths of solutes in drainage water in an urban area during a pluvial flood event. The necessity to capture the complexity of a surface runoff model and the coupling of pipe and surface are investigated. Results show that fully coupled hydrodynamic models are needed for a good representation of transport paths and breakthrough curves of contaminant concentration in drainage water.

Similarly, Mohammadi et al. (2019) [14] studied Contamination transport model by coupling analytical element and point collocation methods. In their study, the ground water flow model is developed by analytical-element method and the contaminant transport model is developed by point collocation method in an unconfined aquifer. Comparison of the model results with the observed data represents a reasonable agreement and capability of the present model in contaminant transport modelling.

Modelling contaminant transport of metal ions through soil was carried out by [1]. The result revealed that the finite difference method using Crank-Nicolson scheme was found to be the most accurate method for estimating the contaminant transport parameters. Tekle and Bisrat (2019) [15] investigated modeling the transport of contaminants in a porous medium applying conservation laws and developing FDMs [based on grids geometry] for solving the resulting PDE. The study established convergence, stability and consistency of the methods. The study has the advantage that water contamination or pollution is related to water quality, climate and aquatic lives such as fishes and food security.

Water resource engineers need reliable support tools for water quality assessment and to predict the effect of their decisions. Many studies have been conducted to test the applicability of the transport models, but none of them have account for variability of dispersion in the model. Hence, existing conventional models used to predict dispersion and transport of contaminants in soil do not account for the variability of dispersion. In this study. The main objective was to investigate the transport of nitrates with dispersion in soil by performing column tests and thereafter solved with Galerkin's finite element method.

2- Materials and Methods

2-1- Description of the Study Area

The study was carried out at Avonkwu Olokoro in Umuahia South, Local Government Area of Abia State. Geographically, Umuahia South L.G.A is located within Lat.5° 26'-5° 34'N and Long.7° 22'-7°33'E within the rain forest belt. The area is characterized by high temperatures of about 29-31°C and has double maxima rainfall peaks in June and July. It is bounded in the north by Umuahia North L.G.A., south by Isiala Ngwa, east by Ikwuano and the west by Imo River which demarcates it with Imo State. Geologically, Umuahia South is within the Benin formation which comprises of shale/sand sediments with intercalation of thin clay beds [16, 17]. Figure 1 shows the Map of Nigeria showing all the states and their capital with the study area outlined; Also, Map of Abia State showing the local Government Areas and Figure 1 showing the rivers and waterways in the basin with their major State capitals and major cities.



Figure 1. Map of the study area.

2-2- Sampling and Analysis

The study adopted experimental research design. Disturbed soil samples were collected from Avonkwu Olokoro in Umuahia South LGA of Abia State using a hand auger at a depth of 6m. The soil samples were air-dried and transferred to a set of sieves mounted on a sieve shaker and vibrated for one hour. The clayey, sandy and silty fractions were then retrieved for characterization. Bulk density, moisture content, hydraulic conductivity, flow rate and flow velocity of the various fractions were determined using BS 1990 and empirical equation. The research was conducted for a period of one year from August 2017 to July 2018. Dispersion studies were performed in the laboratory using soil column filled with silver nitrate (AgNO₃) solution. Samples were collected from the column outlet at intervals of 5minutes and the dispersion coefficient calculated using [18]. Notordamojo's solute transport model was then modified by replacing the constant dispersion coefficient (D) with a variable dispersion coefficient (D') as in Equation 2 obtained using multiple regression.

$$\frac{\partial C}{\partial t} = \frac{D}{R} \frac{\partial^2 C}{\partial z^2} - \frac{V}{R} \frac{\partial C}{\partial z} - \frac{K}{R} C^n \cdot m \cdot t^{m-1}$$
(1)

The modified model was solved using finite element method. Thereafter, the model was verified using the measured data obtained from the study and compared with the existing model.



Figure 2. Flow Chart of the Research Methodology.

2-3-1-D Transport Equation

In order to improve the Equation 1;

$$\frac{\partial C}{\partial t} = \frac{D'}{R} \frac{\partial^2 C}{\partial x^2} - \frac{U}{R} \frac{\partial C}{\partial x} - \frac{K}{R} C^{\alpha} \cdot \beta \cdot t^{\beta - 1} + S_C$$
(2)

Boundary Conditions

$$C = C_0 \text{ at } x = 0 \text{ for all } t$$
$$\frac{\partial C}{\partial x} \to 0 \text{ as } x \to \infty$$
$$C = 0 \text{ at } t = 0 \text{ for all } x$$

2-4- Finite Element Method

Using the finite Element Modeling method is the model improvement of Equation 2;

$$\int_{R} N_{B}L(\phi)dR = 0, \quad \beta = i, j, k$$
(3)

where N_{β} = shape function; \emptyset = uknown parameter, $\emptyset = [N_i, N_j, N_k \dots \dots \dots][\emptyset_p]$, $L(\emptyset)$ = differential eqn. govn. \emptyset , R = region of interest.

$$\frac{\partial C}{\partial t} - \left(\frac{D'}{R}\frac{\partial^2 C}{\partial x^2} - \frac{U}{R}\frac{\partial C}{\partial x} - \frac{K}{R}C^{\alpha}.\beta.t^{\beta-1} + S_C\right) = 0$$
(3)

$$\int_{R} N_{B}L(\phi)dR = 0, \quad \beta = i, j, k$$
(3)

The Equation 5 were discretized into finite element using [19] where the parameters used as defined in section (2.3).

2-5- Galerkin's Finite Element Method

A linear shape function N_k is chosen for this purpose and may be represented as:

$$C(x) = N_i^e C_i + N_{i+1}^e C_{i+1} = [N]\{C\}$$
(6)

In which $N_i^e = 1 - \frac{x}{L}$ and $N_{i+1}^e = \frac{x}{L}$ are the linear shape or approximation functions of element, *e* at node *i* and *i* + 1.

Then, the second step of Derivation of Element equation with the Galerkin's weighted residual method is the basis of the element derivation equation, substituting Equation 6 in Equation 3 we have:

$$\sum_{1}^{K=1} \int_{0}^{L} N^{T} \left[\frac{\partial C}{\partial t} - \frac{D'}{R} \frac{\partial^{2} C}{\partial x^{2}} + \frac{U}{R} \frac{\partial C}{\partial x} + \frac{K}{R} C^{\alpha} \beta t^{\beta-1} - S_{C} \right] dx = 0$$

$$\tag{7}$$

The reduction of the second order term in equation above, to the first order equivalent using integration by parts yields;

$$\sum_{1}^{k=1} \int_{0}^{L} \left[N^{T} * \frac{\partial C}{\partial t} - \frac{D'}{R} * \frac{\partial N^{T}}{\partial x} * \frac{\partial C}{\partial x} + \frac{D'}{R} N^{T} * \frac{\partial C}{\partial x} - \frac{U}{R} * N^{T} * \frac{\partial C}{\partial x} + \frac{K}{R} * N^{T} * C^{\alpha} * \beta * t^{\beta-1} - S_{c} N^{T} \right] dx = 0$$
(8)

Further substitution of the discretized function; $C(x) = [N]{C}$ into Equation 8 gives;

$$\sum_{1}^{k=1} \int_{0}^{L} \left[N^{T} * \frac{\partial}{\partial t} ([N]\{C\}) - \frac{D'}{R} * \frac{\partial N^{T}}{\partial x} * \frac{\partial}{\partial x} ([N]\{C\}) + \frac{D'}{R} N^{T} * \frac{\partial}{\partial x} ([N]\{C\}) - \frac{U}{R} * N^{T} * \frac{\partial}{\partial x} ([N]\{C\}) + \frac{K}{R} * N^{T} * ([N]\{C\}) * \beta * t^{\beta-1} - S_{c} N^{T} \right] = 0$$

$$(9)$$

The integrative evaluation of the individual terms in Equation 9 for a linear element at nodes 1 and 2 with linear shape functions gives;

$$\int_{0}^{T} N^{T} \frac{\partial}{\partial t} ([N] \{C\}) dt = \int_{0}^{T} N^{T} N \frac{\partial L}{\partial t} dt = \int_{0}^{T} \begin{bmatrix} 1 - \frac{t}{T} \\ \frac{t}{T} \end{bmatrix} \left[\left(1 - \frac{t}{T} \right) \quad \frac{t}{T} \right] \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \int_{0}^{T} \begin{bmatrix} \left(1 - \frac{t}{T} \right) \quad \left(1 - \frac{t}{T} \right) \\ \frac{t}{T} \quad \frac{t}{T} \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{1} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left\{ \begin{matrix} C_{2} \\ C_{2} \end{matrix} \right\} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ C_{2} \end{matrix} dt \end{bmatrix} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ C_{2} \end{matrix} dt \end{bmatrix} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ C_{2} \end{matrix} dt \end{bmatrix} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ C_{2} \end{matrix} dt \end{bmatrix} dt \end{bmatrix} dt = \frac{T}{2} \begin{bmatrix} 1 & 1 \\ C_{2} \end{matrix} dt \end{bmatrix} dt \end{bmatrix} dt \end{bmatrix} dt \end{bmatrix} dt \end{bmatrix} dt \end{bmatrix} dt \end{bmatrix}$$

where $C = \frac{\partial C}{\partial t}$, time derivation of C.

$$\int_{0}^{L} N^{T} \frac{\partial}{\partial x} \left([N] \{C\} dx = \int_{0}^{L} N^{T} N \frac{\partial L}{\partial t} dx = \int_{0}^{L} \begin{bmatrix} 1 - \frac{x}{L} \\ \frac{x}{L} \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{cases} C_{1} \\ C_{2} \end{cases} dx = \int_{0}^{L} \begin{bmatrix} \left(1 - \frac{x}{L}\right) & \left(1 - \frac{x}{L}\right) \\ \frac{x}{L} & \left(\frac{x}{L}\right) \end{bmatrix} \begin{cases} C_{1} \\ C_{2} \end{cases} dx = \frac{L}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{cases} C_{1} \\ C_{2} \end{cases} dx = \begin{cases} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{cases} C_{1} \\ C_{2} \end{cases} dx = \begin{cases} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{cases} C_{1} \\ C_{2} \end{cases} dx = \begin{cases} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{cases} C_{1} \\ C_{2} \end{cases} dx = \begin{cases} 1 & 1 \\ C_{2} \end{cases} dx = \begin{cases} 1 & 1 \\ C_{2} \end{cases} dx = \begin{cases} 1 & 1 \\ C_{2} \end{cases} dx = \begin{cases} 1 & 1 \\ C_{2} \end{cases} dx = \begin{cases} 1 & 0 \\ C_{2} \end{cases} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \\ C_{2} \end{bmatrix} dx = \begin{cases} 1 & 0 \\ C_{2} \end{bmatrix} dx = \\$$

Evaluation of second term with linear shape functions:

$$\int_{0}^{L} \frac{\partial N^{T}}{\partial x} \cdot \frac{\partial}{\partial x} ([N]\{C\}) dx = \int_{0}^{L} \begin{bmatrix} -1\\1 \end{bmatrix} \left[\left(1 - \frac{x}{L} \right) \quad \frac{x}{L} \right] \left\{ \frac{C_{1}}{C_{2}} \right\} dx = \int_{0}^{L} \frac{1}{L} \left\{ \frac{-1}{1} \right\} \frac{1}{L} \begin{bmatrix} -1\\1 \end{bmatrix} \left\{ \frac{C_{1}}{C_{2}} \right\} dx = \frac{1}{L} \begin{bmatrix} 1 & -1\\-1 & 1 \end{bmatrix} \left\{ \frac{C_{1}}{C_{2}} \right\}$$
(12)

Evaluation of the third term with linear shape functions:

$$\int_{0}^{L} N^{T}([N]\{C\}) dx = \int_{0}^{L} \left[\frac{1-x}{\frac{L}{x}} \right] \left[\left(1 - \frac{x}{L}\right) \frac{x}{L} \right] \left\{ \begin{array}{c} C_{1} \\ C_{2} \end{array} \right\} dx = \frac{L}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \left\{ \begin{array}{c} C_{1} \\ C_{2} \end{array} \right\}$$
(13)

Evaluation of the fifth term with linear shape function:

$$\int_{0}^{L} N^{T} dx = \int_{0}^{L} S_{c} \begin{bmatrix} (1 - \frac{x}{L}) \\ \frac{x}{L} \end{bmatrix} dx = \frac{L}{2} \begin{cases} 1 \\ 1 \end{cases}$$
(14)

Combining each of the evaluated terms yields the following element equations;

Linear finite element option;

$$\frac{T}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \begin{bmatrix}C_1\\C_2\end{bmatrix} - \frac{D}{R} * \frac{1}{L}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} \begin{bmatrix}C_1\\C_2\end{bmatrix} + \frac{D}{R} * \frac{L}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \begin{bmatrix}C_1\\C_2\end{bmatrix} - \frac{U}{R} * \frac{L}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \begin{bmatrix}C_1\\C_2\end{bmatrix} + \frac{K}{R} * \frac{L}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix} \begin{bmatrix}C_1\\C_2\end{bmatrix} * \beta * t^{\beta-1} - S_c * \frac{L}{2}\begin{bmatrix}1\\1\end{bmatrix}$$
(15)

Rearranging the above equation:

$$\frac{T}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} - \frac{D}{RL} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \frac{DL}{2R} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} - \frac{UL}{2R} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \frac{KL\beta t^{\beta-1}}{6R} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} - \frac{S_c L}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \frac{KL\beta t^{\beta-1}}{6R} \begin{bmatrix} 2 & 1 \\ C_2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} - \frac{S_c L}{2} \begin{bmatrix} 1 \\ C_1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \frac{KL\beta t^{\beta-1}}{6R} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \frac{KL\beta t^{\beta-1}}{2} \begin{bmatrix}$$

2-6- Assemble Element Equations into Global Equation

Taking the constant of left hand matrix as $k_1 = \frac{T}{2}$, $k_2 = \frac{D'}{RL}$, $k_3 = \frac{D'L}{2R}$; $k_4 = \frac{UL}{2R}$ and $k_5 = \frac{KL\beta t^{\beta-1}}{6R}$, But $D' = \frac{e^{6.35 \frac{3}{4}K^{0.411}}}{t^{2.139}*V^{1.143}*d^{0.034}}$ which replaced the constant dispersion coefficient (D) obtained using multiple regression [20].

Then the conductivity matrices to the left of the Equation 16 can be combined as:

$$k_{1} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} C_{1} \\ C_{2} \end{pmatrix} - k_{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} C_{1} \\ C_{2} \end{pmatrix} + k_{3} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} C_{1} \\ C_{2} \end{pmatrix} - k_{4} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} C_{1} \\ C_{2} \end{pmatrix} + k_{5} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{pmatrix} C_{1} \\ C_{2} \end{pmatrix} = \frac{S_{c}L}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{pmatrix} C_{1} \\ C_{2} \end{pmatrix}$$
(17)

Yield the following equation:

$$\begin{bmatrix} (k_1 - k_2 + k_3 - k_4 + 2k_5) & (k_1 + k_2 + k_3 - k_4 + k_5) \\ (k_1 + k_2 + k_3 - k_4 + k_5) & (k_1 - k_2 + k_3 - k_4 + 2k_5) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \frac{S_C L}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
(18)

$$or \ [k]\{C\} = \{V\}$$
(19)

Let
$$B = k_1 - k_2 + k_3 - k_4 + 2k_5$$
 and $D = k_1 + k_2 + k_3 - k_4 + k_5$ (20)

Therefore, the element assemblage result followed the pattern of matrix (21):

гB	D	0	0	0	0	0	0	0	01	$\begin{pmatrix} l_1 \end{pmatrix}$		(1)	
D	2B	D	0	0	0	0	0	0	0	C_2		2	
0	D	2B	D	0	0	0	0	0	0	C_3		2	
0	0	D	2B	D	0	0	0	0	0	C_4		2	
0	0	0	D	2B	D	0	0	0	0	$\int C_5$	$\sum S_{c}L$	2	
0	0	0	0	D	2B	D	0	0	0	C_6	$(-\frac{1}{2})$	2 (
0	0	0	0	0	D	2B	D	0	0	C ₇		2	
0	0	0	0	0	0	D	2B	D	0	C ₈		2	
0	0	0	0	0	0	0	D	2B	D	C9		2	
L0	0	0	0	0	0	0	0	D	B^{\perp}	1C.		いり	

where $2B = 2(k_1 - k_2 + k_3 - k_4 + 2k_5) = 2k_1 - 2k_2 + 2k_3 - 2k_4 + 4k_5$.

3- Results and Discussion

Particle sizes of the soil samples ranged from 0.0002 mm - 0.001 mm, 0.04 mm - 0.25 mm and 0.004 mm-0.006 mm for clayey, sandy and silty soils respectively. The bulk density, moisture content, hydraulic conductivity, rate of flow and flow velocity were obtained as 1.13 g/cm^3 , 20%, $2.7 \times 10^{-5} \text{ cm/s}$, 0.026 m^3 /s, and $1.31 \times 10^{-5} \text{ cm/s}$ respectively for clay. The corresponding values for sand were 1.38 g/cm^3 , 18%, 0.016 cm/s, 0.194 m^3 /s and 0.032 cm/s while those for silt were 1.12 g/cm^3 , 20%, $1.1 \times 10^{-4} \text{ cm/s}$, 0.165 m^3 /s and 1.24 cm/s.

3-1- Model Calibration

Calibration was necessary in other to determine the values of the parameters that can fit the model to the system [21]. In essence of this, half of the experimental data was used for calibration and half used for verification. From the result, both has good agreement with R^2 value of 0.991 as shown in the Figures 2 and 3.



Figure 2. Calibration graph showing predicted and measured of dispersion in the soil.

3-2- Verification of the Model

The verification of the model was done in other determine how well the mathematical model describes the system. This was achieved by conducting a set of measurements from the system used in creating the model. The remaining data were used for verification.

(21)



Figure 3. Verification of graph showing predicted and measured of dispersion in soil.

3-3- Comparison of Modified Notodarmojo and Existing Notodarmojo Model with the Average Measured Values

Figures 4 to 15, shows the measured and predicted models for transport of nitrates in the soil with respect to the distance of the point source (septic tank). The predicted values were lower than the measured values and this is in agreement with the study of [22]. Hence, using the Standard t- test, the predicted values were significant to each other ranging from 0.074-0.331. The difference between the experimental and predicted results, when expressed as a percentage of the experimental values was less than 5% for the modified model. In summary, all the figures are similar in terms of decreases in concentration with increase in distance.



Figure 4. Measured and Predicted Concentrations against Distance for the month of August.

It was observed in Figure 4 that predicted values were lower than the measured values. Also, there was a sharp decrease in nitrate concentrations at 0.561ppm and 0.098 at a distance of 1.8m but slight decreases was observed at different distances and this could be attributed to soil type, adsorption or dispersion in the soil [1]. The correlation coefficient r of the predicted model and measured values P_1 , P_2 were found to be 0.894 and 0.484 respectively.



Figure 5. Measured and predicted Concentrations against Distance for the month of September.

Figure 5 shows that the concentration of nitrate decreases with increase in distance. Also the measured values were lower than predicted values. This variation may be due to the sorption and adsorption which reduces the amount of contaminant transported. The correlation coefficient r of P_1 and P_2 were found to be 0.885 and 0.653.



Figure 6. Measured and Predicted Concentrations against Distance for the month of October.

Figure 6 depicts the measured and predicted concentrations against distance for the month of October, it was observed that the predicted and measured concentration of nitrate at 0.136 and 0.012ppm sharply decreases at a distance of 3.6 m but slight decrease was observed at different distances and this may be related to the absorption of contaminants [23].



Figure 7. Measured and Predicted Concentrations against Distance for the month of November.

Figure 7 shows that the predicted values were lower than the measured values. Also, there was a sharp decrease in nitrate concentrations at 0.156 ppm at a distance of 1.8 m but slight decrease in nitrate at different distances, this could be attributed to soil type and low rainfall intensity in the soil [23]. The correlation coefficient r of P_1 , P_2 were found to be 0.895 and 0.578.



Figure 8. Measured and Predicted Concentrations against Distance for the month of December.

Simulated concentrations of nitrate and distance from point source are depicted in Figure 8. It was observed that predicted values were lower than the measured values. Also, there was a sharp decrease of nitrate concentrations at 0.167 and 0.012 ppm at a distance of 1.8 m for M and P₁ respectively but slight decreases were observed at different distances. This could be attributed to low rainfall intensity in the soil [20]. The correlation r of P₁, P₂ were found to be 0.896 and 0.582 respectively.



Figure 9. Measured and Predicted Concentrations against Distance for the month of January.

It was observed in Figure 9 that predicted values were lower than the measured values. Also, there was a sharp decrease in nitrate concentrations at 0.682 ppm, 0.165 ppm for M and P₁ respectively at a distance of 1.8 m. But there were slight decreases at different distances which this could be attributed to soil type, adsorption or dispersion in the soil [1]. Also, the correlation coefficient r of the modified model P₁ was 0.769 while the experimental data fitted in existing model P₂ was 0.620.



Figure 10. Measured and Predicted Concentrations against Distance for the month of February.

Figure 10 shows the distribution of nitrate concentrations and distance for the month of February. It was observed that there was a rapid decrease in nitrate concentrations at 0.682 ppm, 0.65 ppm for M and P₁ at a distance of 1.8 m but slight decreases in nitrate were observed at different distances. This could be attributed to intrinsic permeability and particle size of the soil. Also, the correlation coefficient r for P₁, P₂ were found to be 0.769 and 0.620 respectively.



Figure 11. Measured and Predicted Concentrations against Distance for the month of March.

Simulated concentrations of nitrate and distance for the month of March are shown in Figure 11. It was observed that predicted values were higher than the measured values. Also, there was a sharp decrease of nitrate concentrations at 0.179 and 0.158 ppm at a distance of 1.8 m for P_1 and M respectively but slight decrease at different distances, this could be attributed to the type of soil or low rainfall intensity in the soil [20]. The correlation coefficient r of the P_1 , P_2 were found to be 0.8886 and 0.579 respectively.



Figure 12. Measured and Predicted Concentrations against Distance for the month of April.

Figure 12 shows that predicted values were slightly higher than the measured values. Also, there was a sharp decrease in nitrate concentrations at 0.171 ppm, 0.167 ppm for P_1 and M respectively at a distance of 1.8 m. There was observed slight decreases at different distances which could be attributed to soil type, adsorption or dispersion in the soil as reported by [1]. Also, the correlation coefficient r of the modified model P_1 , experimental data fitted in existing model P_2 were found to be 0.769 and 0.620 respectively.



Figure 13. Measured and Predicted Concentrations against Distance for the month of May.

Figure 13 shows that the predicted and measured concentration of nitrate of 0.213 and 0.163 ppm sharply decreases at a distance of 1.8 m but slight decreases were observed at different distances. This could be related to the absorption of contaminants [23].



Figure 14. Measured and Predicted Concentrations against Distance for the month of June.

Figure 14 shows that the concentration of nitrate decreases with increase in distance. Also the measured values were observed to be lower than predicted values. This variation may be due to the sorption and dispersion which reduces the amount of contaminant transported. The correlation coefficient r of P_1 , P_2 were found to be 0.8877 and 0.619.



Figure 15. Measured and Predicted Concentrations against Distance for the month of July.

Figure 15 shows that the predicted and measured concentration of nitrate at 0.136 and 0.012 ppm sharply decreases at a distance of 3.6 m but slight decrease was observed at different distances which could be attributed to the absorption of contaminants [23].

4- Conclusion

Dispersion studies performed in the laboratory using column studies to calibrate movement of contaminants in the soil, effectively simulates and predicts the real time phenomenon as evidenced in the statistical verification results. This contaminants in the soil are governed by advection, dispersion, geochemical mass transfer and decay in the case of radioactive materials. The regression dispersion model, (D') was used in modifying a transport model of a nitrate after being linked to [9] and thereafter, solved numerically using Galerkin's Finite Element Method (FEM). Result from this research, is in conformity and measures very well with previous studies. This was demonstrated when the data obtained were fitted into the two models. R² values ranging from 0.741 to 0.896 and 0.484 to 0.769 were observed for the modified model and the existing Notordamojo's model and solving using Finite Element gave better predictions than the existing Notodarmojo's model and solving using Finite Element gave better predictions than the existing Notodarmojo's model and solving using Finite Element gave better predictions than the existing Notodarmojo's model and solving using Finite Element gave better predictions than the existing notodarmojo's model and better prediction of model for transport of nitrates in soils. The study has established that the modified model which accounted for variability in dispersion coefficient offered a better approach than the conventional one.

5- Declarations

5-1-Author Contributions

Conceptualization, A.C. and B.N.; methodology, A.C.; software, A.C; validation, A.C., J.C. and B.N.; formal analysis, A.C.; investigation, A.C.; resources, B.N.; data curation, A.C.; writing—original draft preparation, J.C.; writing—review and editing, J.C.; visualization, J.C.; supervision, J.C.; project administration, J.C.; funding acquisition, B.N. All authors have read and agreed to the published version of the manuscript.

5-2-Data Availability Statement

The data presented in this study are available on request from the corresponding author.

5-3-Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

5-4- Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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